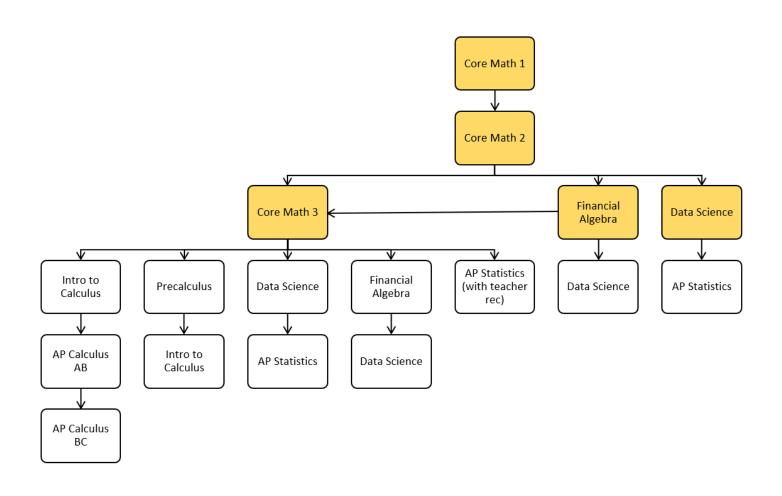


# Math Curriculum

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# Math Sequence:



# Core Math 1

# Core Math 1 Unit 1 Overview

#### Unit Title: Functions

#### **Unit Summary**

This unit explores nonlinear functions. Students learn how to describe functions completely and will investigate the shapes and behaviors of several different nonlinear functions. This chapter also introduces each student to sharing mathematical knowledge with a study team as they work together to solve problems.

### **Approximate Time Needed**

3-4 weeks

Unit Foundation

### **Assessed Competencies**

KC: Functions and Algebra EC: Mathematical Communication

Assessed Standards

 $\label{eq:F-IF} F-IF: \ I \ can \ understand \ the \ concept \ of \ a \ function \ and \ use \ function \ notation.$ 

I can interpret functions in applications in terms of the context.

F-BF: I can build a function that models a relationship between two quantities, with arithmetic sequences. MP 6: Attend to precision

# **Curriculum Framing Questions**

Enduring Understandings

Students will understand ....

how to investigate growth patterns and determine the characteristics of some graphs of nonlinear functions what it means for a relationship to be a function, learn how to use function notation, and determine the domain and range of a function.

how to rewrite expressions with exponents in equivalent, useful forms.

#### **Essential Questions**

Can I identify important quantities in situations and describe their relationships using graphs?

## Core Math 1 Unit 2 Overview

### Unit Title: Unit 2 Linear Functions

#### **Unit Summary**

This unit will focus on the slope and y-intercept of linear functions. Students will look for connections among the multiple representations of linear functions: table, graph, equation, and situation. Also, students explore the idea of slope as a rate of change.

### **Approximate Time Needed**

3-4 weeks

Unit Foundation

Assessed Competencies

- KC: Functions and Algebra
- CT: Modeling and Applications

EC: Mathematical Communication

#### Assessed Standards

A-CED: I can create linear equations that describe numbers or relationships.

- A-REI: I can solve linear equations and inequalities in one variable and explain the reasoning. I can solve systems of linear equations by graphing, substitution, elimination.
- G-GPE: I can analyze slope criteria for parallel and perpendicular lines and use them to solve geometric problems.
- F-LE: I can interpret linear expressions for functions in terms of the situation they model.
- A-CED: I can create a linear model to represent a given situation and solve a problem.

MP6: Attend to precision

#### Curriculum Framing Questions

#### **Enduring Understandings**

Students will understand that...a situation can be represented in multiple ways.

Students will investigate slope as a rate, consider steepness and direction, and create equations given a linear growth situation.

#### **Essential Questions**

Can I create a representation of a problem, consider the units involved, and understand the meaning of the quantities using tables, graphs, and equations?

Unit Title: Transformations and Solving

### **Unit Summary**

This unit students will learn about rigid transformations as they study how to flip, turn, and slide shapes. Then they will learn how to use these transformations to build new shapes and describe symmetry. Students will develop a method to rewrite products of binomials and other polynomials, such as (3x-2)(4+x), using area models. Students will use three methods for solving equations: rewriting, looking inside, and undoing and will develop new ways to solve complicated equations involving multiplication, fractions, and exponents.

# **Approximate Time Needed**

3-4 weeks

Unit Foundation

### **Assessed Competencies**

KC: Functions and Algebra

KC: Geometry

CT: Modeling and Applications

EC: Mathematical Communication

Assessed Standards

G-CO: I can represent rigid transformations in the plane using rotations, translations and reflections. A-SSE: I can interpret algebraic expressions and write them in equivalent forms to solve problems.

# **Curriculum Framing Questions**

Enduring Understandings

Students will understand that...

Rigid transformations maintain a figure's size and shape. Complex expressions and equations can be rewritten to simpler form.

**Essential Questions** 

Can I perform rigid transformations on a shape to create a new shape and describe its symmetry?

Can I rewrite a polynomial using an area model?

Can I solve an equation by rewriting, looking inside, and undoing and will develop new ways to solve complicated equations involving multiplication, fractions, and exponents?

## Unit Title: Modeling Two-Variable Data

#### **Unit Summary**

Students will describe a dependent relationship, called the association, between two numerical variables. Students will use scatterplots of data to create lines and curves that model the data, and then use those models to make predictions. Students will mathematically describe the form, direction, strength, and outliers of an association.

# Approximate Time Needed

3-4 weeks

Unit Foundation

**Assessed Competencies** 

KC: Statistics & Probability CT: Modeling and Applications EC: Mathematical Communication

### Assessed Standards

S-ID: I can summarize and interpret data with multiple representations.

M&A: Stats: I can interpret linear models and distinguish between correlation and causation.

MP3: Construct viable arguments, critique reasoning, support your work

### Curriculum Framing Questions

Enduring Understandings

Students will understand ....

how to "eyeball" a line of best fit and use it to make predictions, interpret the slope and *y*-intercept in a statistical situation, and describe the form, direction, strength, and outliers of an association.

how to calculate residuals and create upper and lower bounds for predictions that are made

how to use a calculator to create the unique line of best fit called the least squares regression line.

how to create residual plots and analyze them to determine whether a model is an appropriate fit to the data and calculate the correlation coefficient and *R*2 and interpret them in context.

**Essential Questions** 

Can I model relationships mathematically in order to describe, analyze, make predictions, and draw conclusions about a set of data?

# Core Math 1 Unit 5 Overview

# Unit Title: Chapter 5 Sequences

### **Unit Summary**

Chapter 5 provides students with an opportunity to review and strengthen your algebra skills while you learn about arithmetic and geometric sequences. Early in the chapter, students will find themselves using familiar strategies such as looking for patterns and making tables to write algebraic equations describing sequences of numbers. Later in the chapter, students will develop shortcuts for writing equations for certain kinds of sequences.

## **Approximate Time Needed**

3 weeks, can be piggy-backed with Chapter 8

Unit Foundation

**Assessed Competencies** 

- KC: Functions and Algebra
- CT: Modeling and Applications

EC: Mathematical Communication

Assessed Standards

A-SSE: I can interpret algebraic expressions and write them in equivalent forms to solve problems. A-CED: I can create linear equations that describe numbers or relationships.

A-CED: I can create exponential equations that describe numbers or relationships.

F-LE: I can interpret exponential expressions for functions in terms of the situation they model.

F-BF: I can build a function that models a relationship between two quantities, with arithmetic sequences.

# Curriculum Framing Questions

**Enduring Understandings** 

Students will learn what sequences are and will become familiar with two important types of sequences: arithmetic and geometric.

Students will write equations for the nth term of arithmetic and geometric sequences.

Students will write recursive equations for sequences, and convert between explicit and recursive equations.. Students will recognize the connections between arithmetic and geometric sequences and linear and exponential functions.

# **Essential Questions**

When patterns are repeated, how can I use the patterns to write equations?

Unit Title: Chapter 6 Systems of Equations

#### **Unit Summary**

This unit explores learning how to solve word problems by writing a pair of equations, called a system of equations. Then students will solve the system of equations using three different methods.

Students will develop ways to solve systems with equations written in different forms, and will learn how to recognize when one method may be more efficient than another.

### **Approximate Time Needed**

2-3 weeks

Unit Foundation

**Assessed Competencies** 

KC: Functions and Algebra CT: Modeling and Applications

EC: Mathematical Communication

Assessed Standards

A-REI: I can solve linear equations and inequalities in one variable and explain the reasoning.

I can solve systems of equations by graphing, substitution, elimination.

F-LE: I can interpret linear expressions for functions in terms of the situation they model.

A-CED: I can create a linear model to represent a situation and solve a problem.

# **Curriculum Framing Questions**

**Enduring Understandings** 

Students will understand ....

how to solve multi-variable equations for one variable and write mathematical sentences (equations) in preparation for solving situational word problems.

how to solve a system of equations in different forms with the Graphing Method

how to solve a system of equations in different forms with the Equal Values and Substitution Methods how to solve a system of equations in different forms using the Elimination Method.

How to determine if a system has one solution, no solution, or infinite solutions

# **Essential Questions**

Is there another approach I can take to help me solve this problem? How else can I look at it?

# Core Math 1 Unit 7 Overview

Unit Title: Chapter 7 Congruence and Coordinate Geometry

#### **Unit Summary**

Students will study congruent shapes—shapes that can be rigidly transformed from one to the other.

Students will explore coordinate geometry.

#### **Approximate Time Needed**

Spring, 3 weeks

Unit Foundation

#### **Assessed Competencies**

KC: Geometry CT: Modeling and Applications EC: Mathematical Communication

### Assessed Standards

G-CO: I can make formal geometric constructions with a variety of tools and methods. I can explain the criteria for triangle congruence.

#### **Curriculum Framing Questions**

Enduring Understandings

Students will review what they know about similar triangles from a previous course, and about rigid transformations from Chapter 3. Students will then develop strategies to justify that two triangles are congruent. They will learn how to use flowcharts to justify conclusions. Students will learn some of the notation used in geometry. Students will focus on congruent triangles and will look for "shortcuts" to determine if two triangles are congruent, and use flowcharts to organize their thinking.

Students will study polygons drawn on coordinate grids. Using algebraic tools, they will justify statements about shapes on coordinate grids, such as " $\Delta ABC$  is a right isosceles triangle." Students will also learn how to find the midpoint of a line segment on a coordinate grid.

# **Essential Questions**

How can I justify that these triangles are congruent?

Unit Title: Chapter 8 Exponential Functions

#### **Unit Summary**

Chapter 8 provides an opportunity for students to learn more about the family of exponential functions. Students will also build more advanced algebra skills, such as writing the equation of an exponential function that passes exactly through any pair of given points.

Students will learn about several important applications of exponential functions and of the intersection of functions.

**Approximate Time Needed** 

Spring, 2-3 weeks; can combine with Chapter 5

Unit Foundation

**Assessed Competencies** 

KC: Functions and Algebra CT: Modeling and Applications EC: Mathematical Communication

Assessed Standards

A-SSE: I can interpret algebraic expressions and write them in equivalent forms to solve problems.

A-CED: I can create linear equations that describe numbers or relationships.

A-CED: I can create exponential equations that describe numbers or relationships.

A-REI: I can solve exponential equations and explain the reasoning.

F-LE: I can interpret exponential expressions for functions in terms of the situation they model.

**Curriculum Framing Questions** 

Enduring Understandings

Students will...Enhance their understanding of exponential functions through multiple representations (tables, graphs, and equations) and applications.

Distinguish between the growth in linear situations and exponential situations.

Model situations using step functions, especially simple and compound interest.

Learn how to graph exponential functions and use them to model everyday situations and solve problems. Learn how to find exponential equations when given two points.

Et an automatical function to contained data and access that fit using re-

Fit an exponential function to scattered data and assess that fit using residual plots.

**Essential Questions** 

What are the connections between the multiple representations and can the student make sense of the situation?

# Core Math 1 Unit 9 Overview

#### Unit Title: Inequalities

#### **Unit Summary**

Students will solve linear and exponential equations for a given variable and represent those solutions graphically. Students will learn how to deal with unequal relationships, called *inequalities*, and will develop ways to represent solutions to inequalities both algebraically and graphically. They will do this for situations involving one variable, two variables, and systems.

Students will extend their ability to work with mathematical sentences by learning how to write mathematical constraints for situations. They will also learn how to work with equations and inequalities involving absolute value.

#### **Approximate Time Needed**

Spring, 2 weeks, omit 9.3

Unit Foundation

#### **Assessed Competencies**

- KC: Functions and Algebra
- CT: Modeling and Applications
- EC: Mathematical Communication

Assessed Standards

A-REI: I can solve linear equations and inequalities in one variable and explain the reasoning. I can solve systems of linear equations by graphing, substitution, elimination.

#### **Curriculum Framing Questions**

**Enduring Understandings** 

How to solve one-variable inequalities and represent the solutions on a number line. Solve one-variable absolute value equations and inequalities.

How to solve two-variable inequalities and how to represent the solutions of two-variable inequalities on an xy-coordinate graph.

Apply what you know about systems of equations to determine the solutions to a system of inequalities.

#### **Essential Questions**

How can I solve the inequality and show all of the solutions?

# Core Math 1 Unit 10 Overview

# Unit Title: Functions and Data

#### **Unit Summary**

Students will start this chapter by looking at two-way tables to determine association of categorical data. Then they will model a golf game and compare results with other teams. Students will review the ways to graphically show data, and decide whether or not to use scatterplots or two histograms to compare two variables. Students will use statistics to compare two sets of data with the center, shape, spread, and outliers. Finally, you will learn a new way to describe the variability (the spread) in data called the standard deviation.

### **Approximate Time Needed**

Late Spring, 2-3 weeks - if time is available

Unit Foundation

### **Assessed Competencies**

KC: Statistics & Probability CT: Modeling and Applications EC: Mathematical Communication

Assessed Standards

S-ID: I can summarize and interpret data with multiple representations.

## **Curriculum Framing Questions**

Enduring Understandings

Students will determine the association of categorical data that is represented on two-way tables. They will review the differences between graphical representations of single-variable data. Students will compare the center, shape, spread, and outliers of two distributions. They will develop a new way to describe the spread called standard deviation.

#### **Essential Questions**

Am I considering all available tools as I approach this problem?

# Core Math 2

# Core Math 2 Unit 1 Overview

Unit Title: Exploring Algebraic and Geometric Relationships

## **Unit Summary**

This unit is designed to expose students to an overview of the concepts they will study in this course, establish classroom norms around working in cooperative learning groups/teams and build their mathematical vocabulary and focus on how to describe relationships using mathematical language.

# **Approximate Time Needed**

Unit 1 - first unit of the year typically takes approximately 2 - 3 weeks.

# Unit Foundation

Assessed Competencies

Knowledge Constructor: Functions & Algebra Knowledge Constructor: Geometry Effective Communicator: Mathematical Communication

Assessed Standards

Algebraic Skills and Reasoning-Quadratic Express.

A-SSE: I can write the area as product and sum. I can rewrite quadratic expressions by completing the square. I can rewrite quadratic expressions by factoring.

A-APR: I can add, subtract, and multiply polynomial expressions.

Geometric Skills and Reasoning-Congruence & Similarity

G-CO: I can understand congruence in terms of rigid transformations. I can determine congruent and supplementary angles with parallel lines cut by a transversal. I can determine relationships between interior and exterior angles of polygons.

G-SRT: I can understand similarity in terms of dilation.

# **Curriculum Framing Questions**

Enduring Understandings

Students will understand that...

Mathematics involves attention to precision and clear communication.

**Essential Questions** 

How can mathematical vocabulary and notation be used to communicate our ideas and thoughts?

Unit Title: Justification and Similarity

## **Unit Summary**

This unit is designed to review methods for proving congruent triangles, and then learning to determine whether two shapes are similar, using both transformations and side and angle relationships. Students will extend their knowledge of proofs to prove that two triangles are similar. Also, they will apply their understanding of similar triangles to solve everyday problems and to prove other geometric relationships.

**Approximate Time Needed** 

Unit 2 - this unit typically takes approximately 3 -4 weeks

Unit Foundation

Assessed Competencies

Knowledge Constructor: Geometry Effective Communicator: Mathematical Communication

Assessed Standards

Geometric Skills and Reasoning-Congruence & Similarity

G-CO: I can understand congruence in terms of rigid transformations. I can determine congruent and supplementary angles with parallel lines cut by a transversal. I can determine relationships between interior and exterior angles of polygons.

G-SRT: I can understand similarity in terms of dilation.

# **Curriculum Framing Questions**

Enduring Understandings

Students will understand that...

Mathematical proof can be used to prove shapes to be congruent and similar. Dilations can be performed and used to prove shapes to be similar. Solve both mathematical and everyday problems involving similar figures and deepen your understanding of proof.

# **Essential Questions**

How can I justify my conclusions?

**Unit Title:** Probability and Trigonometry

# **Unit Summary**

This unit is designed to review methods for proving congruent triangles, and then learning to determine whether two shapes are similar, using both transformations and side and angle relationships. Students will extend their knowledge of proofs to prove that two triangles are similar. Also, they will apply their understanding of similar triangles to solve everyday problems and to prove other geometric relationships.

**Approximate Time Needed** 

Unit 3 - typically takes approximately 3 - 4 weeks.

Unit Foundation

Assessed Competencies

Knowledge Constructor: Geometry Knowledge Constructor: Probability Complex Thinker: Modeling and Application Effective Communicator: Mathematical Communication

Assessed Standards

Geometric Skills and Reasoning-Trigonometry

G-SRT: I can understand trigonometric ratios in order to solve problems involving right triangles. Probability skills and Reasoning

S-CP: I can understand probability models to solve scenarios of unions, intersections, and complements. Modeling and Application - Probability

S-CP: I can understand probability models to solve scenarios of unions, intersections, and complements. S-MD: I can understand expected value to decide fair games and explain how to make a game fair.

# **Curriculum Framing Questions**

Enduring Understandings

Students will understand that...

Several ways to model situations involving probability, such as tree diagrams and area models. Computing probabilities of unions, intersections, and complements of events. Calculate expected value in games of chance. The relationship between the slope of a line and the slope angle. The concept of slope ratio to determine missing measurements of a right triangle and solve everyday problems.

**Essential Questions** 

Can I use the available tools to solve problems and decide which tool might be the most helpful?

# Unit Title: Exploring Algebraic and Geometric Relationships

# **Unit Summary**

This unit is designed to begin to build students' skill at working with quadratic expressions. They have already used area models to represent the area of a rectangle as a product and as a sum. They will learn how to change a quadratic expression written as a sum into its product form. [Section 4.1]

This unit is designed to begin learning two new trigonometric ratios (sine and cosine) and then apply trigonometry to solve problems. [Section 4.2]

# **Approximate Time Needed**

Unit 4 - this unit typically takes approximately 3-4 weeks.

# Unit Foundation

Assessed Competencies

Knowledge Constructor: Functions & Algebra Knowledge Constructor: Geometry Effective Communicator: Mathematical Communication

Assessed Standards

Algebraic Skills and Reasoning-Quadratic Express.

A-SSE: I can write the area as product and sum. I can rewrite quadratic expressions by completing the square. I can rewrite quadratic expressions by factoring.

A-APR: I can add, subtract, and multiply polynomial expressions.

Geometric Skills and Reasoning-Trigonometry

G-SRT: I can understand trigonometric ratios in order to solve problems involving right triangles

# **Curriculum Framing Questions**

# Enduring Understandings

Students will understand that...

Develop a method to change a quadratic expression written as a sum into its product form, also called its factored form. Identify patterns for factoring quadratic expressions. Trigonometric ratios include sine, cosine, and inverse trigonometric functions, and these tools are for solving right triangles. Model situations with right triangles and use trigonometric ratios to solve problems.

#### **Essential Questions**

Can I identify and use structure in the mathematical expression or in the sides or angles of the triangle?

# Core Math 2 Unit 5 Overview

# Unit Title: Quadratic Functions

## **Unit Summary**

This unit is designed to help students make connections between the different representations of a quadratic function. They will generate each representation of a quadratic function (equation, graph, table, and situation) from each of the others to create a quadratic functions web, and they will discover efficient ways to sketch the graph of a parabola. They will also use the methods of factoring and the Zero Product Property, Completing the Square, and the Quadratic Formula to solve quadratic equations. They will study different forms of quadratic equations and see how those forms can be used in everyday situations.

# **Approximate Time Needed**

Unit 5 - this unit typically takes approximately 4 - 5 weeks.

# Unit Foundation

Assessed Competencies

Knowledge Constructor: Functions & Algebra Complex Thinker: Modeling & Application Effective Communicator: Mathematical Communication

#### Assessed Standards

Algebraic Skills/Reasoning-Quadratic Equations

A-REI: I can solve quadratic equations using multiple methods, including factoring, completing the square and quadratic formula.

N-CN: I can solve quadratic equations with real coefficients that have complex solutions.

A-SSE: I can write the area as product and sum. I can rewrite quadratic expressions by completing the square. I can rewrite quadratic expressions by factoring. A-APR: I can add, subtract, and multiply polynomial expressions.

Modeling and Application-Functions and Algebra

F-IF: I can analyze functions in multiple representations (table, graph, equation, situation) and interpret key features.

F-BF: I can graph and write quadratic functions as a transformation of  $y=x^2$ .

# **Curriculum Framing Questions**

#### **Enduring Understandings**

Students will understand that...

Graphs, tables, and equations of quadratic functions to create a quadratic functions web. The Zero Product Property determines the x-intercepts of a parabola. Everyday situations using quadratic functions. Solve quadratic equations by completing the square and by applying the Quadratic Formula. Determine the number of solutions to a quadratic equation and choose a strategy for solving based on the given equation.

# **Essential Questions**

How can I represent a situation mathematically? What different representations can I use?

# Unit Title: More Right Triangles

## **Unit Summary**

This unit is designed to continue the study of the powerful similarity and side ratio relationships in right triangles. Students will explore relationships in special right triangles that can give them more ways to determine missing side and angle measures—even when they do not have a calculator handy!

In addition, Section 6.2 offers several projects and activities that will help them synthesize their understanding and make connections between different concepts they have learned so far. Students will consolidate what they know, apply it in new ways, and identify what they still need to learn.

# **Approximate Time Needed**

Unit 6 - this unit typically takes approximately 3 - 4 weeks.

Unit Foundation

Assessed Competencies

Knowledge Constructor: Functions & Algebra Knowledge Constructor: Geometry Effective Communicator: Mathematical Communication

Assessed Standards

Algebraic Skills/Reasoning-Exponential

F-LE: I can construct and compare linear and exponential models to solve problems.

N-RN: I can extend the properties of exponents to rational exponents to simplify and rewrite expressions. Geometric Skills/Reasoning-Congruence & Similarity

G-CO: I can understand congruence in terms of rigid transformations. I can determine congruent and supplementary angles with parallel lines cut by a transversal. I can determine relationships between interior and exterior angles of polygons.

G-SRT: I can understand similarity in terms of dilation.

Geometric Skills/Reasoning-Trigonometry

G-SRT: Define trigonometric ratios and solve problems involving right triangle

**Curriculum Framing Questions** 

**Enduring Understandings** 

Students will understand that...

Mathematics involves attention to precision and clear communication. The Pythagorean Theorem and properties of similar triangles to discover patterns in special right triangles and those with side lengths that are Pythagorean Triples.

**Essential Questions** 

How can I represent the problem? How does the given information relate to the mathematical ideas I have learned previously?

Unit Title: Proof and Conditional Probability

#### **Unit Summary**

This unit builds on students' knowledge of congruent triangles. They will explore the relationships of the sides and diagonals of a parallelogram, kite, trapezoid, rectangle, and rhombus. As they explore new geometric properties, they will formalize their understanding of proof. [Section 7.1]

Students will use area models and two-way tables to calculate conditional probabilities. They will also learn several ways to determine whether events are independent. [Section 7.2]

# **Approximate Time Needed**

Unit 7 - this unit typically takes approximately 3-4 weeks.

### Unit Foundation

Assessed Competencies

Knowledge Constructor: Statistics & Probability Knowledge Constructor: Geometry Effective Communicator: Mathematical Communication

Assessed Standards

Statistics & Probability

S-CP: I can construct appropriate models to describe events as a subset of sample space (area model, tree diagram).

Geometric Skills/Reasoning-Congruence & Similarity

G-CO: I can understand congruence in terms of rigid transformations. I can determine congruent and supplementary angles with parallel lines cut by a transversal. I can determine relationships between interior and exterior angles of polygons.

G-SRT: I can understand similarity in terms of dilation.

**Curriculum Framing Questions** 

Enduring Understandings

Students will understand that...

Investigate special quadrilaterals by using what congruent triangles can tell you about the relationships of the sides, angles, and diagonals. You will deepen your understanding of proof. Analyze probabilities, develop an understanding of conditional probability and more formal mathematical definitions of independence. Use two-way tables to determine if two categorical variables are associated with each other.

**Essential Questions** 

How can I use the given information to construct arguments, how can I justify my conclusions and how can I respond to the arguments of others?

# Core Math 2 Unit 8 Overview

# Unit Title: Polygons and Circles

#### **Unit Summary**

This unit is designed to expose students to use construction tools and properties of triangles and circles to construct special segments and points. They will apply their knowledge of triangles to make discoveries about the interior and exterior angles of polygons and the areas of regular polygons with five, six, eight, and even 100 sides! Students will re-examine similar shapes to study what happens to the area and perimeter of a shape when the shape is enlarged or reduced. They will then connect their understanding of similar figures and regular polygons to circles and solve problems about length and area.

### **Approximate Time Needed**

Unit 8 - this unit typically takes approximately 2 - 3 weeks.

### Unit Foundation

Assessed Competencies

Knowledge Constructor: Geometry Effective Communicator: Mathematical Communication

Assessed Standards

Geometric Skills/Reasoning-Congruence & Similarity

G-CO: I can understand congruence in terms of rigid transformations. I can determine congruent and supplementary angles with parallel lines cut by a transversal. I can determine relationships between interior and exterior angles of polygons. G-SRT: I can understand similarity in terms of dilation.

**Curriculum Framing Questions** 

Enduring Understandings

Students will understand that...

**Essential Questions** 

How can mathematical vocabulary and notation be used to communicate our ideas and thoughts?

# **Core Math 3**

# Core Math 3 Unit 1 Overview

Unit Title: Investigations and Functions

#### **Unit Summary**

This unit focuses on investigations and justifications in mathematics. This sets the stage for the work that students will do throughout the course where they will be asked to not only solve problems but also investigate patterns and communicate their justifications for their conclusions using formal mathematical notation and language.

**Approximate Time Needed** 

Unit 1 - first unit of the year typically takes approximately 2-3 weeks.

# Unit Foundation

Assessed Competencies

Knowledge Constructor: Functions & Algebra Critical Thinker: Modeling & Applications Effective Communicator: Mathematical Practices

Assessed Standards

KC: Function Skills/Reasoning - Function Families

F-IF - Interpreting functions Interpret and analyze (graph and show key features) Write functions in equivalent forms F-BF - Building functions Transformations (Piecewise, Rational, Absolute Value, Radical)

CT: Modeling & Application - Functions & Algebra

A-CED: Creating equations to model phenomena Rearrange and use them to solve problems.

# EC: Mathematical Practice 3

Construct viable arguments, critique reasoning, support your work

# **Curriculum Framing Questions**

Enduring Understandings

Students will understand the connections between numbers, functions, geometry, algebra and statistics through modeling.

**Essential Questions** 

How do I make sense of and persevere in problem solving to make connections in mathematics?

Unit Title: Transformations of Parent Graphs

#### **Unit Summary**

This unit focuses on generalizations and how to generate families of functions from parent graphs. Students learn how to use transformations to generate and graph a variety of new functions in order to draw conclusions and graph new functions.

# **Approximate Time Needed**

Unit 2 -approximately 5 weeks

Unit Foundation

Assessed Competencies

Knowledge Constructor: Functions & Algebra Critical Thinker: Modeling & Applications Effective Communicator: Mathematical Practices

Assessed Standards

Function Skills/Reasoning - Function Families

F-IF - Interpreting functions Interpret and analyze (graph and show key features) Write functions in equivalent forms

F-BF - Building functions Transformations (Piecewise, Rational, Absolute Value, Radical) Modeling & Application - Functions & Algebra

A-CED: Creating equations to model phenomena Rearrange and use them to solve problems.

Mathematical Practice 3

Construct viable arguments, critique reasoning, support your work

**Curriculum Framing Questions** 

Enduring Understandings

Students will understand that mathematics is the study of patterns and those patterns can be applied to new scenarios. They will also be able to apply horizontal and vertical shifts as well as vertical 'stretch' factors to graphs and equations for the purposes of modeling phenomena.

#### **Essential Questions**

What does the structure of an equation tell me about its graph?

# Core Math 3 Unit 3 Overview

Unit Title: Solving equations, systems of equations and inequalities

#### **Unit Summary**

This unit focuses on the meaning of solutions. Students are asked to determine the number of solutions an equation or inequality may have, as well as solve for those and analyze the meaning of those solutions within an application.

#### Approximate Time Needed

Unit 3 -approximately 5 weeks

#### Unit Foundation

Assessed Competencies

Knowledge Constructor: Functions & Algebra Critical Thinker: Modeling & Applications Effective Communicator: Mathematical Practices

Assessed Standards

KC: Functions and Algebra- AlgebraSkills/Reasoning-Equations Inequalities A-REI: Solving systems algebraically; Solving systems graphically

A-CED: Represent constraints using equations and/or inequalities

CT: Modeling & Applications- Modeling & Application- Geometry

A-CED: Creating equations to model phenomena, rearrange and use them to solve problems.

#### EC: Mathematical Practices

MP-3 Construct viable arguments, critique reasoning, support your work

MP-6 Attend to precision

#### **Curriculum Framing Questions**

Enduring Understandings

Students will understand that...

Mathematics is made up of skills which allow us to solve equations and inequalities and those solutions can be used to solve more complicated applications of mathematics in the real world.

#### **Essential Questions**

How can I find a solution accurately and efficiently? What are the multiple ways to represent solutions to equations, systems and inequalities?

## Unit 4 Title: Normal Distributions and Geometric Modeling

#### **Unit Summary**

This unit focuses on basic techniques of performing opinion surveys along with their limitations and pitfalls. Students will learn why randomness is a cornerstone of statistical studies. They will also create histograms with percentages, called relative frequency histograms. Students will learn a new way to describe the shape of a distribution, and use it to model certain distributions.

In the last section of the unit students will model geometric relationships focused on using 2D shapes to create 3D shapes. They will then summarize and present their work using modeling software.

#### **Approximate Time Needed**

Unit 5 - approximately 5 weeks

#### Unit Foundation

Assessed Competencies

Knowledge Constructor: Statistics & Probability Knowledge Constructor: Geometry Critical Thinker: Modeling & Applications

Assessed Standards

- KC: Statistics and Probability Statistics Skills and Reasoning S-ID: Summarize data in one and two variables Display data in one and two variables Interpret data in one and two variables
- KC: Geometry-Geo. Skills & Reasoning-Measurement & Dimension

G-GMD: Identify cross-sections of 3D objects Predict 3D shapes from 2D revolutions

CT: Modeling & Applications - Statistic & Probability

S-IC: Understand and evaluate random processes underlying statistical experiments; Making inferences and Justifying Conclusions S-ID: Summarize Data in one and two variables; Display data in one and two variables ; Interpreting Data in one and two variables

CT: Modeling & Applications- Modeling & Application- Geometry

G-MG: Apply geometric concepts to model physical objects

EC: Mathematical Practices MP-3 Construct viable arguments, critique reasoning, support your work MP-6Attend to precision

#### **Curriculum Framing Questions**

Enduring Understandings

Students will understand the power and pitfalls of statistical surveys as well as methods to interpret and create data displays. In addition, students will be able to visualize 2D cross-sections of 3D shapes as well as create 3D shapes by transforming 2D shapes.

**Essential Questions** 

What are the powers and pitfalls of statistics? How are 2D and 3D shapes connected in our world?

# Unit 5 Title: Inverses and Logarithms

# **Unit Summary**

In this unit students will extend their understanding of 'undoing' equations to investigate functions that "undo" each other. The focus of the first half will be on investigating inverse relationships and the relationships between functions and their inverses through numeric, algebraic and graphical approaches.

In the second half of the unit, students will determine the inverses of many parent graphs and add them to their tools for working with parent graphs. They will also determine inverses for exponential functions, which are called logarithmic functions and explore their graphs and transformations.

**Approximate Time Needed** 

Unit 5 - approximately 4 weeks

Unit Foundation

Assessed Competencies

Knowledge Constructor: Inverse & Log Functions Effective Communicator: Mathematical Practices

Assessed Standards

KC: Inverse & Log Functions Skills & Reasoning

A-SSE: Simplify expressions using properties of logarithms

F-BF: Verify an inverse function using composition

F-BF: Evaluate inverse function values from a table

F-LE: Solve exponential models by converting to a logarithm and evaluating using technology

#### EC: Mathematical Practices

MP-3 Construct viable arguments, critique reasoning, support your work

MP-6 Attend to precision

**Curriculum Framing Questions** 

Enduring Understandings

Students will understand that an inverse can be found by reversing the process of the original function and can apply this idea fluently to find and test inverses. In addition, students will have an understanding of the exponential and logarithmic graphs as parent functions and are able to extend their ability to transform graphs to these functions.

**Essential Questions** 

What is an inverse and how can it be used?

### **Unit Title:** Logarithms and Triangles

#### **Unit Summary**

In this unit, students will further their understanding of logarithms—which will give them the tools to solve a murder mystery. They will also expand their knowledge of triangle tools to include solving for the sides and the angles of non-right triangles.

Students will return to logarithms to learn more about their properties and why they are useful. They will construct an exponential function to model a situation, and will use logarithms to solve a mathematical murder mystery.

Students will develop tools to calculate the measures of the angles and side lengths of any triangle, provided that enough information is given. They will then explore ways to choose an appropriate tool to solve new problems in unfamiliar contexts.

#### **Approximate Time Needed**

-approximately 5 weeks

## Unit Foundation

Assessed Competencies

Knowledge Constructor: Algebra Skills and Reasoning - Inverse & Log Functions Critical Thinker: Modeling & Applications Effective Communicator: Mathematical Practices

Effective Communicator: Mathematical Prac

Assessed Standards

KC: Algebra Skills and Reasoning - Inverse & Log Functions

A-SSE: Simplify expressions using properties of logarithms F-BF: Verify an inverse function using composition; Evaluate inverse function values from a table

CT: Modeling & Applications: Functions and Algebra

F-LE: Solve exponential models by converting to a logarithm and evaluating using technology

- G-SRT: Understand and apply LoS and LoC
- CT: Modeling & Applications: Geometry
- G-SRT: Understand and apply LoS and LoC
- EC: Mathematical Practices
  - MP3: Construct viable arguments, critique reasoning, support your work
  - MP6: Attend to precision

# **Curriculum Framing Questions**

**Enduring Understandings** 

Students will understand that logarithms and exponential functions have an inverse relationship that can be used to solve problems. Students will understand the format of the Law of Sines and Cosines and be able to apply them to phenomena such as surveying and calculating altitudes.

**Essential Questions** 

How can I use properties of logarithms to predict the results of exponential phenomena? How can I apply trigonometry to solve for immeasurable distances and angles?

# Core Math 3 Unit 7 Overview

## Unit Title: Polynomials

#### **Unit Summary**

In this chapter students will apply their knowledge of families of functions to include polynomial functions. As they investigate the equation  $\leftrightarrow$  graph connection for polynomials, students will learn how to search for factors (which can help them determine x-intercepts) and how to use division to find additional factors.

When they investigate the graphs of polynomial functions and systems involving polynomials, students will see many that appear not to intersect. As students investigate these systems further, they will review their work from a previous course with imaginary and complex numbers.

In the last section of the chapter, students will learn the process of polynomial division and its application to writing equations in factored form.

#### **Approximate Time Needed**

- approximately 4 weeks

#### Unit Foundation

Assessed Competencies

Knowledge Constructor: Functions & Algebra Knowledge Constructor: Geometry Critical Thinker: Modeling & Applications Effective Communicator: Mathematical Practices

Assessed Standards

KC: Functions & Algebra - Polynomial Functions Algebra Skills & Reasoning-Polynomial/Rational

CT: Modeling & Applications - Functions & Algebra

- A-CED: Creating equations to model phenomena, rearrange and use them to solve problems.
- EC: Mathematical Practices
  - MP3: Construct viable arguments, critique reasoning, support your work
  - MP6: Attend to precision

#### **Curriculum Framing Questions**

Enduring Understandings

Students will understand the connections between zeros, factors and x-intercepts and be able to identify behavior of the function based on those features in addition to end behavior. Students will also understand the equivalence of forms and be able to fluidly convert between standard to factored forms.

**Essential Questions** 

What are the ups and downs of polynomials?

## Unit Title: Periodic Functions

#### **Unit Summary**

This chapter begins with students generating a new curve that is a periodic function. They will then explore the relationship between right-triangle trigonometry and this new curve. They will explore and learn the unit circle, a useful tool for working with periodic functions that includes learning radian measurement of angles.

In the second section of this chapter, students will apply their prior knowledge of transforming functions to trigonometric functions. They will add to their knowledge of transformations by learning about a property called the period of the function before learning to write equations for the trigonometric functions.

### **Approximate Time Needed**

-approximately 4 weeks

Unit Foundation

Assessed Competencies

Knowledge Constructor: Functions & Algebra Knowledge Constructor: Geometry Critical Thinker: Modeling & Applications Effective Communicator: Mathematical Practices

Assessed Standards

KC: Functions and Algebra - Periodic Functions

F-IF - Interpreting functions - Interpret and analyze (graph and show key features) and write functions in equivalent forms

F-BF - Building functions Transformations (Sine, Cosine & Tangent)

KC: Geometric Skills and Reasoning - Trigonometry

G-SRT: Apply trig to general triangles

F-TF: Unit Circle, Domain/Range, Periodic phenomena

#### EC: Mathematical Practices

MP3: Construct viable arguments, critique reasoning, support your work

MP6: Attend to precision

**Curriculum Framing Questions** 

Enduring Understandings

Students will understand the connection between right triangles and the trigonometric function. In addition, they will be applying that understanding to evaluate, graph and create equations for trigonometric functions to model periodic phenomena in our world.

**Essential Questions** 

What is a periodic function and where can I apply it in my world?

# Precalculus

# Precalculus Unit 1 Overview

# Unit Title: Algebra - Problem Solving Strategies

### **Unit Summary**

This first unit will set the tone for the year of students taking charge of their learning and working both independently and collaboratively. We will reinforce problem solving strategies learned throughout the core math curriculum and concentrate on the most efficient methods to use in each new situation. Students will also key in on the reasonableness of their solutions and discover the connections between the algebraic and graphical representations. The algebraic functions will include linear, absolute value, rational, radical, rational exponents, exponential and logarithmic. The unit will close by applying these problem-solving skills to new situations and modeling real world phenomena.

# **Approximate Time Needed**

9-10 weeks

# Unit Foundation

# **Assessed Competencies**

Knowledge Constructor: Algebra Critical Thinker: Modeling & Applications Effective Communicator: Mathematical Practices

Assessed Standards

Algebra Skills & Reasoning - Equations & Inequalities Modeling & Applications - Function Analysis Mathematical Practice 3- Construct viable arguments, critique reasoning, support your work

# **Curriculum Framing Questions**

**Enduring Understandings** 

Students will understand that...

- Problems can be solved by multiple methods but efficiency is important
- Real life situations can be modeled and solved both algebraically and graphically
- They should always check the reasonableness of their solutions

#### **Essential Questions**

- How can complex situations be modeled both algebraically and graphically?
- Is there a "right way" to solve a problem?

# **Unit Title: Families of Functions**

## **Unit Summary**

This unit will concentrate on the graphical representations of each of the functions that students worked with in the previous problem-solving unit. Key features to be studied will include x and y intercepts, key points, asymptotes and general shape. Transformations of these functions will also be studied and will include translations, reflections, stretch or compression. These ideas will be extended by looking at combinations and compositions of these functions. The unit will conclude with a graphing project that blends the lines between mathematics and art. Technology will be used extensively during this unit.

**Approximate Time Needed** 

6-7weeks

Unit Foundation

**Assessed Competencies** 

Knowledge Constructor: Functions Critical Thinker: Modeling & Applications Effective Communicator: Mathematical Practices

Assessed Standards

Algebra Skills & Reasoning - Function Families & Graphs Modeling & Applications - Function Analysis Mathematical Practice 3 -Construct viable arguments, critique reasoning, support your work

# **Curriculum Framing Questions**

Enduring Understandings

Students will understand that...

- The beauty of mathematics
- One concept can be applied to many situations (the graphical form of any equation transforms the parent function in the same way)

#### **Essential Questions**

- What are the connections between function's equations and graphs?
- How can I create art using mathematics?

Unit Title: Trigonometry - General Triangles

# **Unit Summary**

In this unit we will extend our knowledge of right triangles to include non-right triangles. This will include the derivation and application of the Laws of Sines and Cosines. We will build on your introduction to these concepts in Core Math 3 and extend that into solving more complex problems. We will conclude the unit by applying these concepts to real world problems.

Approximate Time Needed

3 weeks

Unit Foundation

### **Assessed Competencies**

Knowledge Constructor: Geometry Critical Thinker: Modeling & Applications Effective Communicator: Mathematical Practices

Assessed Standards

Geometric Skills & Reasoning - Trigonometry Modeling & Applications - Trigonometry Mathematical Practice 3 -Construct viable arguments, critique reasoning, support your work

# **Curriculum Framing Questions**

Enduring Understandings

Students will understand ...

- The tools needed to solve triangles (law of sines, law of cosines, right triangle trigonometry, pythagorean theorem, triangle sum theorem)
- How to apply these tools to real world problems

#### **Essential Questions**

How can I use the tools of trigonometry to solve geometric problems?

# Precalculus Unit 4 Overview

# Unit Title: Trigonometric Functions

### **Unit Summary**

This is the biggest unit of the course! We will explore many new aspects of trigonometry and their relationships to each other. Topics include:

- Rotational angles (positive and negative angles, reference angles)
- Find the values of coordinates of key points on the unit circle and see how they can be used to find the values of sine and cosine for any angle.
- Generate parent graphs for sine and cosine and use them to sketch various transformations.
- Define other trigonometric functions in terms of sine and cosine and perform transformations on these new functions.
- Model situations using periodic functions.
- Solving trigonometric equations and use simple trigonometric identities

#### **Approximate Time Needed**

### 10 weeks

# Unit Foundation

### **Assessed Competencies**

Knowledge Constructor: Geometry Critical Thinker: Modeling & Applications Effective Communicator: Mathematical Practices

Assessed Standards

Geometric Skills & Reasoning - Trigonometry Modeling & Applications - Trigonometry Mathematical Practice 3 - Construct viable arguments, critique reasoning, support your work

Curriculum Framing Questions

Enduring Understandings

Students will understand ...

- Rotational angles
- The unit circle generates the graphs of sine and cosine
- The key features of the six trigonometric functions
- Trigonometric expressions can be simplified using the rules of algebra
- Solutions to trigonometric functions
- How to model situations using periodic functions

**Essential Questions** 

How are triangles, circles and periodic functions related? What is trigonometry? How can I use trigonometry to create a symmetric piece of art?

# **Introduction to Calculus**

# Intro to Calculus Unit 1 Overview

Unit Title: Chapter 13: Conic Sections

### **Unit Summary**

In this chapter, students will:

Define and investigate properties of conic sections.

Solve problems involving conic sections.

Learn how to read and work with a college type text.

This unit has two main goals: Work with conic sections and learn how to read and work with a college level mathematics text. This chapter of the book is structured differently than the other chapters to accommodate the second goal.

Approximate Time Needed

This unit takes about 3 weeks to complete

### Unit Foundation

### **Assessed Competencies**

Knowledge Constructor: Geometry Critical Thinker: Modeling & Applications Effective Communicator: Mathematical Practices

Assessed Standards

G-GPE: Expressing Geometric Properties with Equations, MP6: Attend to Precision

**Curriculum Framing Questions** 

Enduring Understandings

Students will understand...

The formal definitions and proofs that generate circles, ellipses, hyperbolas, and parabolas. They will learn how to read through these definitions and proofs and how to approach problems.

**Essential Questions** 

What are the features of a college level mathematics textbook? What are the real-life applications of conic sections?

#### Intro to Calculus Unit 2 Overview

#### Unit Title: Chapter 4: Circular Functions

#### **Unit Summary**

In prior courses, students have worked with trigonometric functions to solve triangles using right triangle trigonometry, the Law of Sines, and the Law of Cosines. In this unit students will find that sine and cosine are also functions. They can be graphed and are useful in many applications. We will develop the trig functions through the use of the unit circle. In this chapter, students will:

Find the values of coordinates of key points on the unit circle and see how they can be used to find the values of sine and cosine for any angle.

Generate parent graphs for sine and cosine and use them to sketch various transformations.

Define other trigonometric functions in terms of sine and cosine.

Model situations using periodic functions.

#### **Approximate Time Needed**

This unit takes about 4 weeks to complete

#### Unit Foundation

#### **Assessed Competencies**

Knowledge Constructor: Geometry Critical Thinker: Modeling & Applications Effective Communicator: Mathematical Practices

Assessed Standards

F-TF: Trigonometric Functions G-SRT: Similarity, Right Triangles and Trig

#### Curriculum Framing Questions

#### Enduring Understandings

Students will understand ..

The unit circle generates the graphs of sine and cosine Trigonometric expressions can be simplified using the rules of algebra

How to model situations using periodic functions

#### **Essential Questions**

How are triangles and circular functions connected? How do trigonometric functions model real-life phenomena? Intro to Calculus Unit 3 Overview

Unit Title: Chapter 5: Introduction to Limits

#### **Unit Summary**

Students have worked with several graphs that have asymptotes. In the case of a horizontal asymptote, a graph approaches a particular value as x gets large. This value is called a limit. In mathematics, the concept of a limit is used to describe the behavior of a function as the independent variable approaches a particular value, or as it becomes arbitrarily large.

We will begin by looking at rational functions, which have both horizontal and vertical asymptotes. We will then look at limits as x approaches a particular value.

In this chapter, students will:

Investigate rational functions and learn how to rewrite such functions in more useful forms. Solve problems involving direct and inverse variation.

Explore how functions behave as x approaches a particular value or goes to infinity.

Learn about one-sided limits and limits of piecewise functions.

Define continuity.

Approximate Time Needed

This unit takes about 4 weeks to complete

### Unit Foundation

**Assessed Competencies** 

Knowledge Constructor: Functions & Algebra Critical Thinker: Modeling & Applications Effective Communicator: Mathematical Practices

Assessed Standards

A-LC: Express and Interpret Limits Symbolically

A-LD1: Estimate limits of functions

A-LD2: Determine limits of functions

# Curriculum Framing Questions

Enduring Understandings

Students will understand...

How to investigate and simplify rational functions.

How variables can be related directly or inversely.

The idea of a reciprocal function.

How to look at limits from several perspectives inc geometry, graphs, tables, and algebra.

The formal definition of continuity.

**Essential Questions** 

How do rational functions and direct and indirect variations connect to the idea of a limit? How can the idea of a limit be represented?

# Intro to Calculus Unit 4 Overview

Unit Title: Chapter 6: Extending Periodic Functions

#### **Unit Summary**

In Unit 2, students worked with transformations of sine and cosine and applied these transformations to a few applications. The main focus of this unit is working with trig equations. Students will be solving trig equations to find when models achieve a desired value. Students will see how the Law of Sines can be used to solve a triangle with more than one solution. You will also develop other trigonometric tools for simplifying expressions using formulas involving sums of angles.

In this chapter, students will:

Solve trigonometric equations. Solve the SSA case of a triangle. Model and solve more complex periodic applications. Simplify expressions involving more than one angle.

Approximate Time Needed

This unit takes about 4 weeks to complete.

Unit Foundation

#### **Assessed Competencies**

Knowledge Constructor: Functions & Algebra Critical Thinker: Modeling & Applications Effective Communicator: Mathematical Practices

Assessed Standards

F-TF: Trigonometric Functions G-SRT: Similarity, Right Triangles and Trig

#### **Curriculum Framing Questions**

Enduring Understandings

Students will understand...

How to solve trig equations by looking at a graph, the unit circle, and using the inverse functions on the calculator. They will see how several trig equations and applications have more than one solution. They will also investigate the inverse trig functions.

How to extend modeling of periodic functions to more complex situations. They will see how a trig function with two different angles can be simplified.

How to work with other trig formulas and identities to solve more complex trig equations

**Essential Question** 

How are cyclic phenomena modeled?

## Intro to Calculus Unit 5 Overview

## Unit Title: Chapter 7, Algebra for College Mathematics

## **Unit Summary**

Mathematics teaches us to think in an organized, logical way and it provides us with powerful problem solving tools. So whether you find yourself in a college math course or in the world of work, the ideas and techniques you will learn in this unit should serve you well.

In this unit you will learn how to:

Describe functions.

Set up, simplify, and solve complex problems.

## **Approximate Time Needed**

This unit takes about two weeks to complete

Unit Foundation

**Assessed Competencies** 

Knowledge Constructor: Functions & Algebra Critical Thinker: Modeling & Applications Effective Communicator: Mathematical Practices

Assessed Standards

A-APR Perform arithmetic operations on Polynomials

F-LE: Linear, Quadratic, Exponential and Logarithmic Models

F-IF : Interpreting Functions

F-BF: Building Functions

## **Curriculum Framing Questions**

**Enduring Understandings** 

Students will understand that...

Functions can be described using specific vocabulary Complex problems can be modeled using algebra

**Essential Questions** 

How can functions be described?

How can complex problems be solved using algebra?

## Intro to Calculus Unit 6 Overview

## Unit Title: Chapter 8: More on Limits

## **Unit Summary**

Students learned about limits back in unit 3 by using a graphical approach. In this unit students will use algebraic techniques to find the limits of functions.

In this chapter, students will:

Use dominant terms to find limits at infinity.

Find limits of rational functions.

Apply exponential functions to real world problems.

**Approximate Time Needed** 

This unit takes about 4 weeks to complete

## Unit Foundation

## **Assessed Competencies**

Knowledge Constructor: Functions & Algebra Critical Thinker: Modeling & Applications Effective Communicator: Mathematical Practices

Assessed Standards

A-LC: Express and Interpret Limits Symbolically

A-LD1: Estimate limits of functions

A-LD2: Determine limits of functions

F-LA: Analyze functions for continuity/discontinuity/differentiability

## **Curriculum Framing Questions**

Enduring Understandings

Students will understand...

How to use algebraic techniques to find limits of a variety of functions. How to use the number e in applications.

**Essential Questions** 

How can limits be used to describe functions? What is the number *e* and what are its applications? Intro to Calculus Unit 7 Overview

Unit Title: Chapter 9: Rates of Change

## **Unit Summary**

In this unit students will investigate rates of change as they occur in the real world. Then students will use limits to determine instantaneous rates of change. We will use what has been learned about limits to define "instantaneous" rates of change. This leads to a crucial relationship between a position function and its related velocity function: a relationship that is fundamental to the study of calculus.

In this unit students will:

Find average rates of change for many different types of functions.

Look at average rates of change, then take limits of these average rates of change to find instantaneous rates of change.

Find ways of relating distance and velocity graphs and distance and velocity functions.

Define the derivative and apply the definition to find the instantaneous rate of change of a function.

## **Approximate Time Needed**

This unit takes about 5 weeks to complete

Unit Foundation

**Assessed Competencies** 

Knowledge Constructor: Functions & Algebra Critical Thinker: Modeling & Applications Effective Communicator: Mathematical Practices

Assessed Standards

A-DC: Determine a derivative using definition A-DD1: Estimate Derivatives A-DD2 Calculate derivatives

## Curriculum Framing Questions

Enduring Understandings

Students will understand ....

How to calculate average rates of change and how to use these averages to estimate instantaneous rates of change.

How to use limits to determine instantaneous rates of change.

How to model position and velocity with graphs and functions

**Essential Questions** 

How can instantaneous rates of change be calculated? How are velocity and position graphs related?

## Intro to Calculus Unit 8 Overview

Unit Title: Rates, Limits, Sums and Continuity

## **Unit Summary**

Students will use limits to determine instantaneous rates of change. In this chapter, students will:

Approximate the area under a curve using Riemann sums and summation notation.

Predict function behavior with limits.

Use limits to define continuity and see how continuity provides the basis for the Intermediate Value Theorem. Develop a method to approximate the velocity of an object at an instant.

Discuss the local linearity of well-behaved functions.

## **Approximate Time Needed**

This unit takes about 4-5 weeks.

## Unit Foundation

## **Assessed Competencies**

Knowledge Constructor: Functions & Algebra Critical Thinker: Modeling & Applications Effective Communicator: Mathematical Practices

Assessed Standards

A-IC: Definite integral as Reimann Sum A-ID1: Calculate definite Integral using areas and properties A-DC: Determine a derivative using definition A-DD1: Estimate Derivatives

## Curriculum Framing Questions

Enduring Understandings

Students will understand...

How to approximate the area under the curve using summation notation and trapezoids.

Limits through approach statements, graphs, and algebra.

Continuity and its relationship with limits.

How to find a method to approximate instantaneous rates of change.

How to use Riemann sums to investigate left endpoint, right endpoint, and midpoint rectangles to approximate the area under a curve.

**Essential Questions** 

How can the area under a curve be found? What is continuity?

## Intro to Calculus Unit 9 Overview

## Unit Title: Slope and Curve Analysis

## **Unit Summary**

Studying rates of change also raises some new areas of focus. How is the rate of change changing? Is the rate of change for all functions the same? How can we generalize the rate of change of a function at an instant? In this chapter, students will:

Find slope functions for many Parent Graphs both graphically and analytically.

Derive and use the formal definition of a derivative as the limit of the slope of a secant line.

Find derivatives of sine, cosine, and formalize the Power Rule.

Discover what 1st and 2nd indicate about a function's shape, including where it is increasing, decreasing and its concavity.

Sketch f'(x) and f''(x) from f(x)

Connect derivatives and second derivatives with velocity and acceleration.

Antidifferentiate.

Investigate and categorize functions that are not differentiable everywhere.

## **Approximate Time Needed**

This unit takes about 4-5 weeks. - If there is time in the year- otherwise covered in a college level calculus course.

## Unit Foundation

## **Assessed Competencies**

Knowledge Constructor: Functions & Algebra Critical Thinker: Modeling & Applications Effective Communicator: Mathematical Practices

## Assessed Standards

F-DA1: Use derivatives to analyze properties of a function; position and velocity graphs, curve sketching

## **Curriculum Framing Questions**

## Enduring Understandings

Students will understand ...

How to use the Power Rule of differentiation.

How to formalize a definition of instantaneous rates of change (IROC).

How to use the definition of the derivative to differentiate sinusoidal functions.

How to sketch functions and their 1st and 2nd derivatives.

How to determine conditions under which a function is and is not differentiable at a point.

**Essential Questions** 

How can I find the first and second derivative of a function?

What does it mean for a function to be differentiable?

How are derivatives connected to the concepts of velocity and acceleration?

## **Financial Algebra**

## Financial Algebra Unit 1 Overview

Unit Title: Chapter 1: The Stock Market

## **Unit Summary**

The goal of this unit is for students to develop a basic understanding of a stock market and to be able to describe stock transactions and trends numerically, graphically and verbally. In this chapter, students will:

Understand different types of business organizations

Understand the purpose of a stock market

Read and interpret stock market data

Represent stock market data with charts and graphs

Calculate net and percent gains and losses that result from stock trades

**Approximate Time Needed** 

This unit takes approximately 4-5 weeks to complete.

## Unit Foundation

## **Assessed Competencies**

Knowledge Constructor: Functions & Algebra Effective Communicator: Mathematical Communication

Assessed Standards

Quantitative Reasoning, Algebraic Skills and Reasoning - Linear, MP6: Attend to Precision

## Curriculum Framing Questions

Enduring Understandings

Students will understand that...

A stock exchange is a marketplace where investors buy and sell shares of ownership in a company

Buying and selling stock is a way of investing money

There are risks and rewards involved in trading stocks

There are a number of investment products available including bonds and mutual funds

Mathematics is key to understanding stock markets and to making investment decisions

**Essential Questions** 

How can I use mathematics to understand stock markets?

## Financial Algebra Unit 2 Overview

Unit Title: Chapter 2: Modeling a Business

## **Unit Summary**

The goal of this unit is for students to be able to mathematically model a business by creating and graphing expense, revenue and profit equations. The students will analyze the equations and graphs in order to determine break-even points and maximum profits. In this chapter, students will:

Create and interpret lines of best fit Analyze supply and demand curves Create and graph expense and revenue functions Determine break even points Create and graph profit functions Determine maximum profit from a profit function

#### **Approximate Time Needed**

This unit takes 4-5 weeks

## Unit Foundation

## **Assessed Competencies**

Knowledge Constructor: Functions & Algebra Knowledge Constructor: Stats & Probability Critical Thinker: Modeling & Applying Effective Communicator: Mathematical Communication

Assessed Standards

Algebraic Skills and Reasoning - Linear, Quantitative Reasoning, M&A: Linear, M&A: Stats

## Curriculum Framing Questions

## Enduring Understandings

Students will understand that...

Data analysis is an important component of designing a business Supply and demand are key concepts is business Fixed and variable expenses are important concepts in business Expenses, revenues and profit can be modeled with functions and graphs Break-even points and maximum profits can be calculated and visualized using functions and graphs

**Essential Questions** 

How can I model a business using mathematics?

## Financial Algebra Unit 2 Overview

Unit Title: Chapter 3: Banking Services

## **Unit Summary**

One goal for this unit includes learning about a variety of types of banking services such as checking accounts and savings accounts. Another goal is for students to understand that compound interest can be modeled with exponential functions. In this chapter, students will:

Understand that financial institutions offer a variety of products

Understand that compound interest is modeled with exponential functions

Calculate the values of investments using exponential functions

Use the number e is used to model continuous growth

**Approximate Time Needed** 

This unit takes 3-4 weeks

Unit Foundation

**Assessed Competencies** 

Knowledge Constructor: Functions & Algebra Critical Thinker: Modeling & Applying Effective Communicator: Mathematical Communication

Assessed Standards

Algebraic Skills and Reasoning- Exponential, Quantitative Reasoning, M&A: Exponential, MP6: Attend to Precision

## **Curriculum Framing Questions**

Enduring Understandings

Students will understand that...

Financial institutions provide a variety of products

Understand that compound interest is modeled with exponential functions

Mathematics can be used to make financial decisions regarding financial products

**Essential Questions** 

How can I use mathematics to understand the services provided by financial institutions?

## Financial Algebra Unit 4 Overview

## Unit Title: Chapter 4: Consumer Credit

## **Unit Summary**

Two goals for this unit are for students to understand that there are costs associated with borrowing money and that there are a variety of institutions that provide credit. In this chapter, students will:

Calculate the finance charges/interest involved with loans

Understand the relationship between loan length and interest paid

Calculate loan length given a monthly payment

Understand how credit cards work and how to calculate the finance charge on a credit card

**Approximate Time Needed** 

This unit takes about 3-4 weeks

Unit Foundation

**Assessed Competencies** 

Knowledge Constructor: Functions & Algebra Critical Thinker: Modeling & Applying Effective Communicator: Mathematical Communication

Assessed Standards

Algebraic Skills and Reasoning- Exponential, Quantitative Reasoning, M&A: Exponential, MP6: Attend to Precision

**Curriculum Framing Questions** 

Enduring Understandings

Students will understand that...

There are costs associated with borrowing money

There is an inverse relationship between loan length and the cost of a loan.

It is important to understand how credit cards work and what sorts of fees are involved with them

**Essential Questions** 

How can I use mathematics to understand consumer credit?

## Financial Algebra Unit 5 Overview

Unit Title: Chapter 5: Automobile Ownership

## **Unit Summary**

The goal of this unit is to have students develop an understanding of the various financial aspects of owning an automobile. In this chapter, student will:

Use statistics to examine reasonable prices for automobiles

Understand that there are a variety of types of automobile insurance

Calculate the costs of automobile insurance

Model automobile depreciation with linear and exponential functions

Use mathematics to understand aspects of driving safety and of crash investigation

**Approximate Time Needed** 

This unit takes 5-6 weeks to complete

## Unit Foundation

## **Assessed Competencies**

Knowledge Constructor: Functions & Algebra Knowledge Constructor: Stats & Probability Critical Thinker: Modeling & Applying Effective Communicator: Mathematical Communication

Assessed Standards

Algebraic Skills and Reasoning - Linear, Algebraic Skills and Reasoning- Exponential, Quantitative Reasoning, Data Analysis, M&A: Linear, M&A: Exponential, M&A: Stats, MP6: Attend to Precision

**Curriculum Framing Questions** 

Enduring Understandings

Students will understand that...

Owning an automobile is a responsibility to be takes seriously and has a number of costs involved Statistics can be used to determine fair buying/selling prices for cars

Automobile insurance has several components

Automobiles tend to depreciate in value of time and mathematics can be used to model this depreciation Mathematics can be used to understand a number of safety issues involved with owning/operating a car

**Essential Questions** 

How can I use mathematics to help me understand the impacts of owning an automobile?

## Financial Algebra Unit 6 Overview

Unit Title: Chapter 6: Employment Basics

#### **Unit Summary**

A goal of this chapter is for students to learn about the variety of ways that employees may receive compensation and benefits. Another goal is for students to understand the structure and purposes of Social Security and Medicare. In this chapter, students will:

Calculate compensation for a variety of employment scenarios

Calculate the value of employee benefits

Model Social Security and Medicare expenses with piecewise functions

## **Approximate Time Needed**

This unit takes 2 to 3 weeks.

## Unit Foundation

#### **Assessed Competencies**

Knowledge Constructor: Functions & Algebra Critical Thinker: Modeling & Applying Effective Communicator: Mathematical Communication

Assessed Standards

Algebraic Skills and Reasoning - Linear, Quantitative Reasoning, M&A: Linear, MP6: Attend to Precision

## **Curriculum Framing Questions**

Enduring Understandings

Students will understand that...

Employees are paid in a variety of ways How to understand and calculate the value of benefits Social Security and Medicare are insurance programs that Americans both pay for and depend on

## **Essential Questions**

How can mathematics help me understand employment basics?

## Financial Algebra Unit 7 Overview

**Unit Title:** Chapter 7: Income Taxes

## **Unit Summary**

The goals for this unit is for students to understand how federal income taxes are calculated and to understand the components of a federal tax return. In this chapter, students will:

Calculate federal taxes owed using tax schedules and tables

Model US income taxes with piecewise functions

Calculate Social Security and Medicare payments for a variety of employment scenarios

**Approximate Time Needed** 

This unit takes about 3 weeks to complete

Unit Foundation

**Assessed Competencies** 

Knowledge Constructor: Functions & Algebra Critical Thinker: Modeling & Applying Effective Communicator: Mathematical Communication

Assessed Standards

Algebraic Skills and Reasoning - Linear, Quantitative Reasoning, M&A: Linear, MP6: Attend to Precision

Curriculum Framing Questions

Enduring Understandings

Students will understand that...

The US has a progressive tax system

How to calculate income tax owed given a tax schedule

The importance of employee benefits when taking a job

Social Security and Medicare are insurance programs that are designed to help older Americans or Americans in need

**Essential Questions** 

How can mathematics help me understand the US federal income tax code?

## Financial Algebra Unit 8 Overview

## Unit Title: Introduction to Budgeting

#### **Unit Summary**

The goal of this unit is for students to create and balance a salary-based budget. In this unit, students will: Use the normal model to understand the range of expenses in a budget Use linear regression to make predictions about the value of expenses in a budget Create a salary-based budget based on research of current economic trends

**Approximate Time Needed** 

This unit takes 2-3 weeks

Unit Foundation

#### **Assessed Competencies**

Knowledge Constructor: Functions & Algebra Knowledge Constructor: Stats & Probability Critical Thinker: Modeling & Applying Effective Communicator: Mathematical Communication

Assessed Standards

Algebraic Skills and Reasoning - Linear, Quantitative Reasoning, Data Analysis, M&A: Linear, M&A: Stats, MP6: Attend to Precision

## **Curriculum Framing Questions**

**Enduring Understandings** 

Students will understand that... Budgeting in an important life skill Mathematics plays an integral part in the creation of a budget

**Essential Questions** 

How can mathematics help me create a realistic salary-based budget?

## **AP Calculus AB**

## AP Calculus AB Unit 1 Overview

Unit Title: Limits and Continuity

## **Unit Summary**

Limits introduce the subtle distinction between evaluating a function at a point and considering what value the function is approaching, if any, as x approaches a point. This distinction allows us to extend understanding of asymptotes and holes in graphs with formal definitions of continuity. We will review rational functions when introducing limits. Limits are the foundation for differentiation (Unit 2), integration (Unit 6). They are the basis for important definitions and for theorems that are used to solve realistic problems involving change and to justify conclusions.

**Approximate Time Needed** 

Approximately 2 -3 weeks

Unit Foundation

**Assessed Competencies** 

Knowledge Constructor: Derivatives and Differentiation Effective Communicator: Math Communication

Assessed Standards

LIM-1 Reasoning with definitions, theorems, and properties can be used to justify claims about limits.

LIM-1.A Represent limits analytically using correct notation.

LIM-1.B Interpret limits expressed in analytic notation.

LIM-1.C Estimate limits of functions.

LIM-1.D Determine the limits of functions using limit theorems.

LIM-1.E Determine the limits of functions using equivalent expressions for the function or the squeeze theorem.

## LIM-2 Reasoning with definitions, theorems, and properties can be used to justify claims about continuity.

LIM-2.A Justify conclusions about continuity at a point using the definition.

LIM-2.B Determine intervals over which a function is continuous.

LIM-2.C Determine values of x or solve for parameters that make discontinuous functions continuous, if possible. LIM-2.D Interpret the behavior of functions using limits involving infinity.

FUN-1 Existence theorems allow us to draw conclusions about a function's behavior on an interval without precisely locating that behavior.

FUN-1.A Explain the behavior of a function on an interval using the Intermediate Value Theorem.

## **Curriculum Framing Questions**

Enduring Understandings

Students will understand that...

that change occur at an instant

that knowing the value of a limit, or that a limit does not exist, helps you to make sense of interesting features of functions and their graphs

how to close loopholes so that a conclusion about a function is always true?

**Essential Questions** 

Introducing Calculus: Can Change Occur at an Instant?

## AP Calculus AB Unit 2 Overview

## Unit Title: Differentiation - Definition of the Derivative

## **Unit Summary**

Derivatives allow us to determine instantaneous rates of change. To develop understanding of how the definition of the derivative applies limits to average rates of change, create opportunities for students to explore average rates of change over increasingly small intervals. Graphing calculator explorations of how various operations affect slopes of tangent lines help students to make sense of basic rules and properties of differentiation.

## Students will be able to

- **General Estimate a derivative using average rate of change**
- □ Calculate the instantaneous rate of change at a point using limits
- □ Calculate a derivative function using limits
- Use continuity and discontinuity to analyze differentiability

### **Approximate Time Needed**

1-2 weeks

## Unit Foundation

## **Assessed Competencies**

Knowledge Constructor: Derivatives and Differentiation Effective Communicator: Math Communication

Assessed Standards

CHA-2 Derivatives allow us to determine rates of change at an instant by applying limits to knowledge about rates of change over intervals.

CHA-2.A Determine average rates of change using difference quotients.

CHA-2.B Represent the derivative of a function as the limit of a difference quotient.

CHA-2.D Estimate derivatives.

FUN-2 Recognizing that a function's derivative may also be a function allows us to develop knowledge about the related behaviors of both.

FUN-2.A Explain the relationship between differentiability and continuity

**LIM-3 Reasoning with definitions, theorems, and properties can be used to determine a limit.** LIM-3.A Interpret a limit as a definition of a derivative.

## **Curriculum Framing Questions**

Enduring Understandings

Students will understand that...

Limits can be used to find instantaneous rates of change

We can find slope values for non-linear functions.

How mathematical properties and rules for simplifying and evaluating limits apply to differentiation

**Essential Questions** 

How do we find rates of change at an instant in time?

## AP Calculus AB Unit 3 Overview

## Unit Title: Differentiation and Differentiation Techniques

#### **Unit Summary**

In this unit, students will discover techniques for differentiation that will allow them to find the derivative of a wide variety of functions. We encourage students to apply the order of operations as they select differentiation rules. In this unit, students learn how to differentiate composite functions using the chain rule and apply that understanding to determine derivatives of implicit and inverse functions. Students need to understand that for composite functions, y is a function of u while u is a function of x. Mastering the chain rule is essential to success in all future units. Developing differentiation skills will allow students to model realistic instantaneous rates of change in Unit 4 and to analyze graphs in Unit 5. Students will be able to

- □ Use the Power Rule, Exponential Rule, Product & Quotient Rules, Chain Rule, Trigonometric Rules, Inverse Rule to calculate the derivative function
- □ Use Implicit Rules to calculate the derivative function for implicitly defined functions
- □ Calculate higher order derivatives

## **Approximate Time Needed**

Approximately 6 weeks

## Unit Foundation

#### **Assessed Competencies**

Knowledge Constructor: Derivatives and Differentiation Effective Communicator: Math Communication

Assessed Standards

# CHA-2 Derivatives allow us to determine rates of change at an instant by applying limits to knowledge about rates of change over intervals.

CHA-2.C Determine the equation of a line tangent to a curve at a given point.

## FUN-3 Recognizing opportunities to apply derivative rules can simplify differentiation.

FUN-3.A Calculate derivatives of familiar functions.

FUN-3.B Calculate derivatives of products and quotients of differentiable functions.

FUN-3.C Calculate derivatives of compositions of differentiable functions.

FUN-3.D Calculate derivatives of implicitly defined functions.

FUN-3.E Calculate derivatives of inverse and inverse trigonometric functions.

FUN-3.F Determine higher order derivatives of a function.

## **Curriculum Framing Questions**

Enduring Understandings

Students will understand that...

Derivatives allow us to determine rates of change at an instand

Algebra and simplification allows us to determine techniques for differentiation of all known functions.

**Essential Questions** 

How do we find instantaneous rates of change for non-linear functions without needing technology (in other words, how do we find exact values for instantaneous rates of change)?

## AP Calculus AB Unit 4 Overview

## Unit Title: Contextual Applications of Differentiation

## **Unit Summary**

Unit 4 begins by developing understanding of average and instantaneous rates of change in problems involving motion. The unit then identifies differentiation as a common underlying structure on which to build understanding of change in a variety of contexts. Students' understanding of units of measure often reinforces their understanding of contextual applications of differentiation. In problems involving related rates, identifying the independent variable common to related functions may help students to correctly apply the chain rule.

Students will be able to

- □ Use derivatives to explain the relationships between position, velocity and acceleration
- □ Use derivatives to describe the relationship between rates in an applied context

## **Approximate Time Needed**

2 - 3 weeks

Unit Foundation

**Assessed Competencies** 

Critical Thinker: Modeling and Applications Effective Communicator: Math Communication

Assessed Standards

CHA-3 Derivatives allow us to solve real-world problems involving rates of change

CHA-3.A Interpret the meaning of a derivative in context.

CHA-3.B Calculate rates of change in applied contexts.

CHA-3.C Interpret rates of change in applied contexts.

CHA-3.D Calculate related rates in applied contexts

CHA-3.E Interpret related rates in applied contexts.

**Curriculum Framing Questions** 

Enduring Understandings

Students will understand that...

Derivatives allow us to solve real-world problems involving rates of change

**Essential Questions** 

What do we know if we know a derivative? What does a derivative mean?

## AP Calculus AB Unit 5 Overview

## Unit Title: Analytic Applications of Differentiation

## **Unit Summary**

In this unit, the superficial details of contextual applications of differentiation are stripped away to focus on abstract structures and formal conclusions. Reasoning with definitions and theorems establishes that answers and conclusions are more than conjectures; they have been analytically determined. As when students showed supporting work for answers in previous units, students will learn to present justifications for their conclusions about the behavior of functions over certain intervals or the locations of extreme values or points of inflection. The unit concludes this study of differentiation by applying abstract reasoning skills to justify solutions for realistic optimization problems. Students will be able to

- **Use derivatives to predict behaviors and determine extrema on functions**
- □ Use derivatives to determine extrema in applied contexts
- □ Use derivatives to evaluate limits of indeterminate forms

## **Approximate Time Needed**

2 - 4 weeks (usually spans over winter break in December)

## Unit Foundation

## **Assessed Competencies**

Critical Thinker: Modeling and Applications Effective Communicator: Math Communication

Assessed Standards

# FUN-1 Existence theorems allow us to draw conclusions about a function's behavior on an interval without precisely locating that behavior.

FUN-1.B Justify conclusions about functions by applying the Mean Value Theorem over an interval.

FUN-1.C Justify conclusions about functions by applying the Extreme Value Theorem

## FUN-4 A function's derivative can be used to understand some behaviors of the function.

FUN-4.A Justify conclusions about the behavior of a function based on the behavior of its derivatives.

FUN-4.B Calculate minimum and maximum values in applied contexts or analysis of functions

FUN-4.C Interpret minimum and maximum values calculated in applied contexts.

FUN-4.E Justify conclusions about the behavior of an implicitly defined function based on evidence from its derivatives.

**Curriculum Framing Questions** 

**Enduring Understandings** 

Students will understand that...

Existence theorems allow us to draw conclusions about a function's behavior on an interval without precisely locating that behavior.

A function's derivative can be used to understand, predict and justify behaviors of the function.

**Essential Questions** 

How are derivatives used in the "real world"? Why do functions behave the way they do?

## AP Calculus AB Unit 6 Overview

## Unit Title: Integration and Accumulation of Change

## **Unit Summary**

This unit establishes the relationship between differentiation and integration using the Fundamental Theorem of Calculus. Students explore the contextual meaning of areas of certain regions bounded by rate functions. Integration determines accumulation of change over an interval. Students should understand that integration is a limiting case of a sum of products (areas) in the same way that differentiation is a limiting case of a quotient of differences (slopes). Future units will apply the idea of accumulation of change to a variety of realistic and geometric applications. Students will be able to

- □ Approximate the area under a curve using limits
- Use the FTC to evaluate definite integrals and calculate values on the antiderivative (and graph)
- **Use applicable properties to calculate definite integrals**
- **C**reate and evaluate integral defined functions, including calculating their derivatives
- □ Use derivative rules and substitution to create antiderivatives

## Approximate Time Needed

Approximately 4 weeks

## Unit Foundation

## **Assessed Competencies**

Knowledge Constructor: Integrals and Integration Effective Communicator: Math Communication

Assessed Standards

CHA-4 Definite integrals allow us to solve problems involving the accumulation of change over an interval.

CHA-4.A Interpret the meaning of areas associated with the graph of a rate of change in context.

- LIM-5 Definite integrals can be approximated using geometric and numerical methods
  - LIM-5.A Approximate a definite integral using geometric and numerical methods.
  - LIM-5.B Interpret and represent the limiting case of the Riemann sum as a definite integral.
- FUN-5 The Fundamental Theorem of Calculus connects differentiation and integration.
- FUN-5.A Represent accumulation functions using definite integrals.

**FUN-6 Recognizing opportunities to apply knowledge of geometry and mathematical rules can simplify integration.** FUN-6.A Calculate a definite integral using areas and properties of definite integrals.

FUN-6.B Evaluate definite integrals analytically using the Fundamental Theorem of Calculus.

FUN-6.C Determine antiderivatives of functions and indefinite integrals, using knowledge of derivatives.

FUN-6.D For integrands requiring substitution or rearrangements into equivalent forms: (a) Determine indefinite integrals. (b) Evaluate definite integrals.

## Curriculum Framing Questions

**Enduring Understandings** 

Students will understand that...Definite integrals allow us to solve problems involving the accumulation of change over an interval. The Fundamental Theorem of Calculus connects differentiation and integration.

## **Essential Questions**

## AP Calculus AB Unit 7 Overview

Unit Title: Differential Equations

## **Unit Summary**

In this unit, students will learn to set up and solve separable differential equations. Slope fields can be used to represent solution curves to a differential equation and build understanding that there are infinitely many general solutions to a differential equation, varying only by a constant of integration. Students can locate a unique solution relevant to a particular situation, provided they can locate a point on the solution curve. By writing and solving differential equations leading to models for exponential growth and decay.

Students will be able to

- □ Solve differential equations for general and specific form
- Use derivatives to create slope fields
- **Use slope fields to model antiderivatives and estimate values of a function**

## **Approximate Time Needed**

2 - 3 weeks

Unit Foundation

**Assessed Competencies** 

Knowledge Constructor: Integrals and Integration Effective Communicator: Math Communication

Assessed Standards

## FUN-7 Solving differential equations allows us to determine functions and develop models.

FUN-7.A Interpret verbal statements of problems as differential equations involving a derivative expression

FUN-7.B Verify solutions to differential equations.

FUN-7.C Estimate solutions to differential equations.

FUN-7.D Determine general solutions to differential equations.

FUN-7.E Determine particular solutions to differential equations.

FUN-7.F Interpret the meaning of a differential equation and its variables in context.

FUN-7.G Determine general and particular solutions for problems involving differential equations in context.

**Curriculum Framing Questions** 

Enduring Understandings

Students will understand that...

Solving differential equations allows us to determine functions and develop models.

**Essential Questions** 

If we know the rate of change of a quantity, can we determine the original function?

## AP Calculus AB Unit 8 Overview

## Unit Title: Applications of Integration

## **Unit Summary**

In this unit, students will learn how to find the average value of a function, model particle motion and net change, and determine areas & volumes by the graphs of functions. Emphasis should be placed on developing an understanding of integration that can be transferred across these and many other applications. Understanding that the area and volume problems studied in this unit are limiting cases of Riemann sums of rectangle areas or prism volumes saves students from memorizing a long list of seemingly unrelated formulas and develops meaningful understanding of integration. Students will be able to

- □ Use integrals to calculate average value a function
- □ Use integrals to calculate area between curves
- □ Use integrals to calculate volumes of solids

## **Approximate Time Needed**

## 2 - 3 weeks

## Unit Foundation

## **Assessed Competencies**

Critical Thinker: Modeling & Applications Effective Communicator: Math Communication

Assessed Standards

#### **CHA-4 Definite integrals allow us to solve problems involving the accumulation of change over an interval.** CHA-4.B Determine the average value of a function using definite integrals.

CHA-4.C Determine values for positions and rates of change using definite integrals in problems involving

rectilinear motion. CHA-4.D Interpret the meaning of a definite integral in accumulation problems.

CHA-4.E Determine net change using definite integrals in applied contexts.

CHA-5 Definite integrals allow us to solve problems involving the accumulation of change in area or volume over an interval.

CHA-5.A Calculate areas in the plane using the definite integral.

CHA-5.B Calculate volumes of solids with known cross sections using definite integrals.

CHA-5.C Calculate volumes of solids of revolution using definite integrals.

## **Curriculum Framing Questions**

**Enduring Understandings** 

Students will understand that...

Definite integrals allow us to solve problems involving the accumulation of change over an interval. Definite integrals allow us to solve problems involving the accumulation of change in area or volume over an interval.

**Essential Questions** 

How do we extend geometry to non-linear shapes and surfaces?

## AP Calculus BC Unit 1 Overview

**Unit Title:** Limits and Continuity

## **Unit Summary**

This unit hones the subtle distinction between evaluating a function at a point and considering what value the function is approaching, if any, as x approaches a point. This distinction allows us to extend understanding of asymptotes and holes in graphs with formal definitions of continuity. Limits are the foundation for differentiation (Unit 2), integration (Unit 6), and infinite series (Unit 10) and are the basis for important definitions and for theorems that are used to solve realistic problems involving change and to justify conclusions.

## **Approximate Time Needed**

1-2 weeks

## Unit Foundation

## **Assessed Competencies**

Knowledge Constructor: Derivatives and Differentiation Effective Communicator: Math Communication

Assessed Standards

## LIM-1 Reasoning with definitions, theorems, and properties can be used to justify claims about limits.

- Represent limits analytically using correct notation.
- Interpret limits expressed in analytic notation.
- Estimate limits of functions.
- Determine the limits of functions using limit theorems.
- Determine the limits of functions using equivalent expressions for the function or the squeeze theorem.

## LIM-2 Reasoning with definitions, theorems, and properties can be used to justify claims about continuity.

- Justify conclusions about continuity at a point using the definition.
- Determine intervals over which a function is continuous.
- Determine values of x or solve for parameters that make discontinuous functions continuous, if possible.
- Interpret the behavior of functions using limits involving infinity.
- FUN-1 Existence theorems allow us to draw conclusions about a function's behavior on an interval without precisely locating that behavior.
  - Explain the behavior of a function on an interval using the Intermediate Value Theorem.

## Curriculum Framing Questions

## Enduring Understandings

Students will understand that...

- Change can occur at an instant.
- Knowing the value of a limit, or that a limit does not exist, can help them to make sense of interesting features of functions and their graphs.
- They can guarantee a conclusion about a function is always true by closing 'loopholes'.

Essential Question

Why are limits and continuity integral to understanding calculus?

## AP Calculus AB Unit 2/3 Overview

Unit Title: Differentiation: Definition, Fundamental Properties and Rules

## **Unit Summary**

Derivatives allow us to determine instantaneous rates of change. Students will strengthen their understanding of how the definition of the derivative applies limits to average rates of change, explore average rates of change over increasingly small intervals and review techniques for differentiation that will allow them to find the derivatives of a broad variety of functions. Honing these skills will allow students to model realistic rates of change in Unit 4 and to analyze graphs in Unit 5.

**Approximate Time Needed** 

3-4 weeks

Unit Foundation

**Assessed Competencies** 

Knowledge Constructor: Derivatives and Differentiation Effective Communicator: Math Communication

Assessed Standards

CHA-2 Derivatives allow us to determine rates of change at an instant by applying limits to knowledge about rates of change over intervals.

- Determine average rates of change using difference quotients.
- Represent the derivative of a function as the limit of a difference quotient.
- Determine the equation of a line tangent to a curve at a given point.
- Estimate derivatives.

FUN-2 Recognizing that a function's derivative may also be a function allows us to develop knowledge about the related behaviors of both.

- Explain the relationship between differentiability and continuity
- LIM-3 Reasoning with definitions, theorems, and properties can be used to determine a limit.
  - Interpret a limit as a definition of a derivative.
- FUN-3 Recognizing opportunities to apply derivative rules can simplify differentiation.
  - Calculate derivatives of familiar functions.
  - Calculate derivatives of products and quotients of differentiable functions.
  - Calculate derivatives of compositions of differentiable functions.
  - Calculate derivatives of implicitly defined functions.
  - Calculate derivatives of inverse and inverse trigonometric functions.
  - Determine higher order derivatives of a function.

## **Curriculum Framing Questions**

Enduring Understandings

Students will understand that...

- They can use derivatives to calculate instantaneous rates of change.
- They can apply algebraic techniques and simplification to differentiate any known function.

**Essential Questions** 

How can I calculate the exact instantaneous rate of change for any function without the need for technology?

## AP Calculus BC Unit 4 Overview

## Unit Title: Contextual Applications of Differentiation

## **Unit Summary**

Unit 4 continues to deepen students' understanding of average and instantaneous rates of change in problems involving motion. The unit then identifies differentiation as a common underlying structure on which to build understanding of change in a variety of contexts. Students' understanding of units of measure then reinforces their understanding of contextual applications of differentiation. In problems involving related rates, identifying the independent variable common to related functions will help students to correctly apply the chain rule. When applying differentiation to determine limits of certain indeterminate forms using L'Hospital's rule, students will first show that the rule applies.

## **Approximate Time Needed**

1-2 weeks

Unit Foundation

## **Assessed Competencies**

Critical Thinker: Modeling and Applications Effective Communicator: Math Communication

Assessed Standards

## CHA-3 Derivatives allow us to solve real-world problems involving rates of change

- Interpret the meaning of a derivative in context.
- Calculate rates of change in applied contexts.
- Interpret rates of change in applied contexts.
- Calculate related rates in applied contexts
- Interpret related rates in applied contexts.

## **Curriculum Framing Questions**

**Enduring Understandings** 

Students will understand that...

• Derivatives allow them to solve real-world applications involving rates of change.

**Essential Questions** 

What is a derivative's meaning in a given situation and what information can it tell me?

## AP Calculus BC Unit 5 Overview

## Unit Title: Analytical Applications of Differentiation

## **Unit Summary**

In this unit, students focus on abstract structures and formal conclusions of differentiation without contextual information. Reasoning with definitions and theorems establishes that answers and conclusions are more than conjectures; they have been analytically determined. As when students showed supporting work for answers in previous units, students will learn to present justifications for their conclusions about the behavior of functions over certain intervals or the locations of extreme values or points of inflection. The unit concludes this study of differentiation by applying abstract reasoning skills to justify solutions for realistic optimization problems.

## **Approximate Time Needed**

2-3 weeks (timed with previous units to end before Thanksgiving break)

## Unit Foundation

## **Assessed Competencies**

Critical Thinker: Modeling and Applications Effective Communicator: Math Communication

Assessed Standards

# FUN-1 Existence theorems allow us to draw conclusions about a function's behavior on an interval without precisely locating that behavior.

- Justify conclusions about functions by applying the Mean Value Theorem over an interval.
- Justify conclusions about functions by applying the Extreme Value Theorem

## FUN-4 A function's derivative can be used to understand some behaviors of the function.

- Justify conclusions about the behavior of a function based on the behavior of its derivatives.
- Calculate minimum and maximum values in applied contexts or analysis of functions
- Interpret minimum and maximum values calculated in applied contexts.
- Justify conclusions about the behavior of an implicitly defined function based on evidence from its derivatives.

**Curriculum Framing Questions** 

Enduring Understandings

Students will understand that...

- They can conclude information about a function's behavior without actually calculating that behavior by applying theorems of existence (MVT & EVT).
- They can apply a function's derivative to understand, justify and predict behaviors of the function.

**Essential Questions** 

How can I apply derivatives in the "real world"? Why do functions behave as they do?

## AP Calculus BC Unit 6 Overview

Unit Title: Integration and Accumulation of Change

## **Unit Summary**

This unit focuses on having students expand their integration techniques from the previous course to include Integration by Parts, Trigonometric Techniques and Partial Fractions. It also continues the relationship between differentiation and integration using the Fundamental Theorem of Calculus with students reviewing the contextual meaning of areas of certain regions bounded by rate functions. Future units will apply the idea of accumulation of change to a variety of realistic and geometric applications.

**Approximate Time Needed** 

3-4 weeks (usually spanning Winter break)

Unit Foundation

**Assessed Competencies** 

Knowledge Constructor: Integrals and Integration Effective Communicator: Math Communication

Assessed Standards

- CHA-4 Definite integrals allow us to solve problems involving the accumulation of change over an interval.
  Use appropriate units of measure.
- LIM-5 Definite integrals can be approximated using geometric and numerical methods
  - Approximate a definite integral using geometric and numerical methods.
- FUN-5 The Fundamental Theorem of Calculus connects differentiation and integration.
  - Represent accumulation functions using definite integrals.
- FUN-6 Recognizing opportunities to apply knowledge of geometry and mathematical rules can simplify integration.
  - Calculate a definite integral using areas and properties of definite integrals.
  - Evaluate definite integrals analytically using the Fundamental Theorem of Calculus.
  - Determine antiderivatives of functions and indefinite integrals, using knowledge of derivatives.
  - Apply appropriate mathematical rules or procedures, with and without technology, to: (a) Determine indefinite integrals. (b) Evaluate definite integrals.
  - Identify an appropriate mathematical rule or procedure based on the classification of a given expression (e.g., Use the chain rule to find the derivative of a composite function).

## **Curriculum Framing Questions**

Enduring Understandings

Students will understand that...

- The Fundamental Theorem of Calculus connects differentiation and integration.
- Definite integrals allow them to solve applications involving accumulation of change over an interval.
- There are many algebraic approaches to finding an antiderivative and some will be more efficient than others.

**Essential Questions** 

Can I find the antiderivative of any function and, if so, which method would be most efficient?

## AP Calculus BC Unit 7 Overview

## Unit Title: Differential Equations

## **Unit Summary**

In this unit, students will learn to set up and solve separable differential equations. Slope fields will be reviewed as general solutions to a differential equation, varying only by a constant of integration. Students will then concentrate on Euler's Method to approximate solutions numerically and Separation of Variables to to find both general and particular solutions. Students will then work to write and solve differential equations leading to models for exponential growth and decay as well as logistic growth.

## **Approximate Time Needed**

2-3 weeks

Unit Foundation

## **Assessed Competencies**

Critical Thinker: Modeling & Applications Effective Communicator: Math Communication

Assessed Standards

## FUN-7 Solving differential equations allows us to determine functions and develop models.

- Interpret verbal statements of problems as differential equations involving a derivative expression
- Verify solutions to differential equations.
- Estimate solutions to differential equations.
- Determine general solutions to differential equations.
- Determine particular solutions to differential equations.
- Interpret the meaning of a differential equation and its variables in context.
- Determine general and particular solutions for problems involving differential equations in context.

## Curriculum Framing Questions

**Enduring Understandings** 

Students will understand that...

• Solving differential equations allows us to determine functions and develop models.

## **Essential Questions**

Can I determine the original function from only a rate of change?

## AP Calculus BC Unit 8 Overview

## Unit Title: Applications of Integration

## **Unit Summary**

In this unit, students will review and extend their ability to find the average value of a function, model particle motion and net change, and determine areas and volumes defined by the graphs of functions. Additionally, they will learn how to calculate arc length and surface areas defined by the graphs of functions. Understanding that the area, volume, and length problems studied in this unit are limiting cases of Riemann sums of rectangle areas, prism volumes, or segment lengths saves students from memorizing a long list of seemingly unrelated formulas and develops meaningful understanding of integration.

## **Approximate Time Needed**

2-3 weeks

Unit Foundation

**Assessed Competencies** 

Critical Thinker: Modeling & Applications Effective Communicator: Math Communication

Assessed Standards

# CHA-4 Definite integrals allow us to solve problems involving the accumulation of change over an inter val.

- Determine the average value of a function.
- Determine values for positions and rates of change in problems involving rectilinear motion.
- Interpret the meaning of a definite integral in accumulation problems.
- Determine net change in applied contexts.

# CHA-5 Definite integrals allow us to solve problems involving the accumulation of change in area or volume over an interval.

- Calculate areas in the plane using the definite integral.
- Calculate volumes of solids with known cross sections using definite integrals.
- Calculate volumes of solids of revolution using definite integrals.
- CHA-6 Definite integrals allow us to solve problems involving the accumulation of change in length or surface area over an interval.
  - Calculate the length of planar curve defined by a function using a definite integral.
  - Calculate the surface area of a planar curve rotated about an axis using a definite integral.

## **Curriculum Framing Questions**

## Enduring Understandings

Students will understand that...

- Definite integrals allow us to solve problems involving the accumulation of change over an interval.
- Definite integrals allow us to calculate lengths, areas, volumes and surface areas by accumulating change.

**Essential Questions** 

How can I extend integration to calculate geometric dimensions of non-linear curves and solids?

## AP Calculus BC Unit 9 Overview

## Unit Title: Parametric Equations, Polar Coordinates and Vector-Valued Functions

## **Unit Summary**

Students will build on their understanding of straight-line motion to solve problems in which particles are moving along curves in the plane. Students will define parametric equations and vector-valued functions to describe planar motion and apply calculus to solve motion problems. Students will learn that polar equations are a special case of parametric equations and will apply calculus to analyze graphs and determine lengths and areas. This unit should be treated as an opportunity to reinforce past learning and transfer knowledge and skills to new situations, rather than as a new list of facts or strategies to memorize.

- **Approximate Time Needed**
- 2-3 weeks

Unit Foundation

**Assessed Competencies** 

Knowledge Constructor: Parametric, Polar and Vector-Valued Functions Critical Thinker: Modeling and Applications Effective Communicator: Math Communication

Assessed Standards

- CHA-3 Derivatives allow us to solve real-world problems involving rates of change.
  - Calculate derivatives of parametric functions.
  - Calculate derivatives of vector-valued functions.
- CHA-5 Definite integrals allow us to solve problems involving the accumulation of change in area or volume over an interval.
  - Calculate areas of regions defined by polar curves using definite integrals.
- CHA-6 Definite integrals allow us to solve problems involving the accumulation of change in length or surface area over an interval.
  - Determine the length of a curve in the plane defined by parametric functions, using a definite integral.
- FUN-3 Recognizing opportunities to apply derivative rules can simplify differentiation.
  - Calculate derivatives of functions written in polar coordinates.
- FUN-8 Solving an initial value problem allows us to determine an expression for the position of a particle moving in the plane.
  - Determine a particular solution given a rate vector and initial conditions.
  - Determine values for positions and rates of change in problems involving planar motion.

## **Curriculum Framing Questions**

Enduring Understandings

Students will understand that...

• Non-linear motion can be analyzed using the chain rule and existing understandings of differentiation and integration.

**Essential Questions** 

• How can I model and analyze motion that is not constrained to a linear path?

## AP Calculus BC Unit 10 Overview

Unit Title: Infinite Sequences and Series

## **Unit Summary**

In this unit, students need to understand that a sum of infinitely many terms may converge to a finite value. They can develop intuition by exploring graphs, tables, and symbolic expressions for series that converge and diverge and for Taylor polynomials. Students should build connections to past learning, such as how evaluating improper integrals relates to the integral test or how using limiting cases of power series to represent continuous functions relates to differentiation and integration. Students who rely solely on memorizing a long list of tests and procedures typically find little success achieving a lasting conceptual understanding of series.

## **Approximate Time Needed**

4-5 weeks (should conclude before April break, but could extend one additional week)

## Unit Foundation

## **Assessed Competencies**

Knowledge Constructor: Series Effective Communicator: Math Communication

Assessed Standards

## LIM-7 Applying limits may allow us to determine the finite sum of infinitely many terms.

- Determine whether a series converges or diverges.
- Approximate the sum of a series.

## LIM-8 Power series allow us to represent associated functions on an appropriate interval.

- Represent a function at a point as a Taylor polynomial.
- Approximate function values using a Taylor polynomial.
- Determine the error bound associated with a Taylor polynomial approximation.
- Determine the radius of convergence and interval of convergence for a power series.
- Represent a function as a Taylor series or a Maclaurin series.
- Interpret Taylor series and Maclaurin series.
- Represent a given function as a power series.

## **Curriculum Framing Questions**

## Enduring Understandings

Students will understand that...

- The sum of infinitely many discrete terms may have a finite value.
- Continuous functions can be represented by the sum of infinitely many discrete terms.

## **Essential Questions**

How can the sum of an infinite series be finite? How can I integrate (exactly or approximately) *any* function?

## **AP Statistics**

## AP Statistics Unit 1 Overview

Unit Title: Exploring and Understanding Data

## **Unit Summary**

Unit 1 introduces students to data and the vocabulary of statistics. Students also learn to talk about data in real-world contexts. Variability in data may seem to suggest certain conclusions about the data distribution, but not all variation is meaningful. Statistics allows us to develop shared understandings of uncertainty and variation. In this unit, students will define and represent categorical and quantitative variables, describe and compare distributions of one-variable data, and interpret statistical calculations to assess claims about individual data points or samples. Students will also begin to apply the normal distribution model as an introduction to how theoretical models for populations can be used to describe some distributions of sample data. Later units will more fully develop probabilistic modeling and inference.

## **Approximate Time Needed**

Students will have to start this unit over the summer July to early September. In theory it would take about 12 class periods. This unit consists mostly of prior learning that it is completed over the summer.

Unit Foundation

**Assessed Competencies** 

Knowledge Constructor: Data Analysis Knowledge Constructor: Probability and Simulation Critical Thinker: Statistical Argumentation Effective Communicator: Mathematical Communication

Assessed Standards

- Describe data presented numerically or graphically.
- Construct numerical or graphical representations of distributions.
- Calculate summary statistics, relative positions of points within a distribution, correlation, and predicted response.
- Compare distributions or relative positions of points within a distribution.
- Determine relative frequencies, proportions, or probabilities using simulation or calculations.
- Interpret statistical calculations and findings to assign meaning or assess a claim.

## **Curriculum Framing Questions**

Enduring Understandings

Students will understand that...

Given that variation may be random or not, conclusions are uncertain.

Graphical representations and statistics allow us to identify and represent key features of data. The normal distribution can be used to represent some population distributions.

**Essential Questions** 

What Can We Learn from Data?

- Is my cat old, compared to other cats?
- How certain are we that what seems to be a pattern is not just a coincidence?

## AP Statistics Unit 2 Overview

Unit Title: Exploring Two-Variable Data

## **Unit Summary**

Students will explore relationships in two-variable categorical or quantitative data sets. They will use graphical and numerical methods to investigate an association between two categorical variables. Skills learned while working with two-way tables will transfer to calculating probabilities in Unit 4. Students will describe form, direction, strength, and unusual features for an association between two quantitative variables. They will assess correlation and, if appropriate, use a linear model to predict values of the response variable from values of the explanatory variable. Students will interpret the least-squares regression line in context, analyze prediction errors (residuals), and explore departures from a linear pattern.

**Approximate Time Needed** 

Mid September to mid/late October 4-5 weeks

## Unit Foundation

#### **Assessed Competencies**

Knowledge Constructor: Data Analysis Critical Thinker: Statistical Argumentation Effective Communicator: Mathematical Communication

Assessed Standards

- Describe data presented numerically or graphically.
- Construct numerical or graphical representations of distributions.
- Calculate summary statistics, relative positions of points within a distribution, correlation, and predicted response.
- Compare distributions or relative positions of points within a distribution.
- Interpret statistical calculations and findings to assign meaning or assess a claim.

**Curriculum Framing Questions** 

Enduring Understandings

Students will understand that...

Given that variation may be random or not, conclusions are uncertain.

Graphical representations and statistics allow us to identify and represent key features of data. Regression models may allow us to predict responses to changes in an explanatory variable.

**Essential Questions** 

Does the fact that the number of shark attacks increases with ice cream sales necessarily mean that ice cream sales cause shark attacks?

How might you represent incomes of individuals with and without a college degree to help describe similarities and/or differences between the two groups?

How can you determine the effectiveness of a linear model that uses the number of cricket chirps per minute to predict temperature?

## AP Statistics Unit 3 Overview

## Unit Title: Collecting Data

## **Unit Summary**

Depending on how data are collected, we may or may not be able to generalize findings or establish evidence of causal relationships. For example, if random selection is not used to obtain a sample from a population, bias may result and statistics from the sample cannot be assumed to generalize to the population. For data collected using well-designed experiments, statistically significant differences between or among experimental treatment groups are evidence that the treatments caused the effect. Students learn important principles of sampling and experimental design in this unit; they will learn about statistical inference in Units 6–9.

## **Approximate Time Needed**

4-5 weeks, mid/late October to Thanksgiving

## Unit Foundation

## **Assessed Competencies**

Knowledge Constructor: Data Analysis Knowledge Constructor: Selecting Statistical Methods Critical Thinker: Statistical Argumentation Effective Communicator: Mathematical Communication

Assessed Standards

- Identify key and relevant information to answer a question or solve a problem.
- Describe an appropriate method for gathering and representing data.
- Make an appropriate claim or draw an appropriate conclusion.
- Interpret statistical calculations and findings to assign meaning or assess a claim.

**Curriculum Framing Questions** 

**Enduring Understandings** 

Students will understand that...

Given that variation may be random or not, conclusions are uncertain. The way we collect data influences what we can and cannot say about a population. Well-designed experiments can establish evidence of causal relationships.

## **Essential Questions**

What does our data tell us? Why might the data we collected not be valid for drawing conclusions about an entire population?

## AP Statistics Unit 4 Overview

Unit Title: Probability, Random Variables, and Probability Distributions

## **Unit Summary**

Probabilistic reasoning allows statisticians to quantify the likelihood of random events over the long run and to make statistical inferences. Simulations and concrete examples can help students to understand the abstract definitions and calculations of probability. This unit builds on understandings of simulated or empirical data distributions and fundamental principles of probability to represent, interpret, and calculate parameters for theoretical probability distributions for discrete random variables. Interpretations of probabilities and parameters associated with a probability distribution should use appropriate units and relate to the context of the situation.

**Approximate Time Needed** 

5-6 weeks, End of November to early January

## Unit Foundation

Assessed Competencies

Knowledge Constructor: Data Analysis Knowledge Constructor: Probability and Simulation Critical Thinker: Statistical Argumentation Effective Communicator: Mathematical Communication

Assessed Standards

- Construct numerical or graphical representations of distributions.
- Determine relative frequencies, proportions, or probabilities using simulation or calculations.
- Determine parameters for probability distributions.
- Describe probability distributions.
- Interpret statistical calculations and findings to assign meaning or assess a claim.

Curriculum Framing Questions

Enduring Understandings

Students will understand that...

Given that variation may be random or not, conclusions are uncertain.

Simulation allows us to anticipate patterns in data.

The likelihood of a random event can be quantified.

Probability distributions may be used to model variation in populations.

Probabilistic reasoning allows us to anticipate patterns in data.

**Essential Questions** 

How can an event be both random and predictable? About how many rolls of a fair six-sided die would we anticipate it taking to get three 1s?

## AP Statistics Unit 5 Overview

## Unit Title: Sampling Distributions

#### **Unit Summary**

This unit applies probabilistic reasoning to sampling, introducing students to sampling distributions of statistics they will use when performing inference in Units 6 and 7. Students should understand that sample statistics can be used to estimate corresponding population parameters and that measures of center (mean) and variability (standard deviation) for these sampling distributions can be determined directly from the population parameters when certain sampling criteria are met. For large enough samples from any population, these sampling distributions can be approximated by a normal distribution. Simulating sampling distributions helps students to understand how the values of statistics vary in repeated random sampling from populations with known parameters.

**Approximate Time Needed** 

3 weeks, Beginning of January

## Unit Foundation

#### **Assessed Competencies**

Knowledge Constructor: Probability and Simulation Critical Thinker: Statistical Argumentation Effective Communicator: Mathematical Communication

Assessed Standards

- Determine relative frequencies, proportions, or probabilities using simulation or calculations.
- Determine parameters for probability distributions.
- Describe probability distributions.
- Interpret statistical calculations and findings to assign meaning or assess a claim.

**Curriculum Framing Questions** 

**Enduring Understandings** 

Students will understand that...

*Given that variation may be random or not, conclusions are uncertain. The normal distribution may be used to model variation. Probabilistic reasoning allows us to anticipate patterns in data.* 

**Essential Questions** 

How likely is it to get a value this large just by chance? How can we anticipate patterns in the values of a statistic from one sample to another?

## AP Statistics Unit 6 Overview

Unit Title: Inference for Categorical Data: Proportions

#### **Unit Summary**

This unit introduces statistical inference, which will continue through the end of the course. Students will analyze categorical data to make inferences about binomial population proportions. Provided conditions are met, students will use statistical inference to construct and interpret confidence intervals to estimate population proportions and perform significance tests to evaluate claims about population proportions. Students begin by learning inference procedures for one proportion and then examine inference methods for a difference between two proportions. They will also interpret the two types of errors that can be made in a significance test, their probabilities, and possible consequences in context.

#### **Approximate Time Needed**

6-7 weeks, January to early March

## Unit Foundation

#### **Assessed Competencies**

Knowledge Constructor: Probability and Simulation Knowledge Constructor: Selecting Statistical Methods Critical Thinker: Statistical Argumentation Effective Communicator: Mathematical Communication

Assessed Standards

- Identify an appropriate inference method for confidence intervals.
- Identify null and alternative hypotheses.
- Identify an appropriate inference method for significance tests.
- Construct a confidence interval, provided conditions for inference are met.
- Calculate a test statistic and find a p-value, provided conditions for inference are met.
- Make an appropriate claim or draw an appropriate conclusion.
- Interpret statistical calculations and findings to assign meaning or assess a claim.
- Verify that inference procedures apply in a given situation.
- Justify a claim based on a confidence interval.
- Justify a claim using a decision based on significance tests.

## **Curriculum Framing Questions**

**Enduring Understandings** 

Students will understand that...

Given that variation may be random or not, conclusions are uncertain.

An interval of values should be used to estimate parameters, in order to account for uncertainty. The normal distribution may be used to model variation.

Significance testing allows us to make decisions about hypotheses within a particular context. Probabilities of Type I and Type II errors influence inference.

**Essential Questions** 

What questions will foster inquiry, understanding, and transfer of learning? When can we use a normal distribution to perform inference calculations involving population

## proportions?

How can we narrow the width of a confidence interval?

If the proportion of subjects who experience serious side effects when taking a new drug is smaller than the proportion of subjects who experience serious side effects when taking a placebo, how can we determine if the difference is statistically significant?

## AP Statistics Unit 7 Overview

Unit Title: Inference for Quantitative Data: Means

#### **Unit Summary**

In this unit, students will analyze quantitative data to make inferences about population means. Students should understand that t\* and t -tests are used for inference with means when the population standard deviation, gives a slightly different value, , is not known. Using s for in the formula for t, whose distribution, which depends on sample size, has more area in the tails than a normal distribution. The boundaries for rejecting a null hypothesis using a z t -distribution tend to be further from the mean than for a normal distribution. Students should understand how and why conditions for inference with proportions and means are similar and different.

#### **Approximate Time Needed**

4-5 weeks, March

## Unit Foundation

#### **Assessed Competencies**

Knowledge Constructor: Probability and Simulation Knowledge Constructor: Selecting Statistical Methods Critical Thinker: Statistical Argumentation Effective Communicator: Mathematical Communication

Assessed Standards

- Identify an appropriate inference method for confidence intervals.
- Identify null and alternative hypotheses.
- Identify an appropriate inference method for significance tests.
- Construct a confidence interval, provided conditions for inference are met.
- Calculate a test statistic and find a p-value, provided conditions for inference are met.
- Make an appropriate claim or draw an appropriate conclusion.
- Interpret statistical calculations and findings to assign meaning or assess a claim.
- Verify that inference procedures apply in a given situation.
- Justify a claim based on a confidence interval.
- Justify a claim using a decision based on significance tests.

## **Curriculum Framing Questions**

**Enduring Understandings** 

Students will understand that...

*Given that variation may be random or not, conclusions are uncertain.* 

*The t-distribution may be used to model variation.* 

An interval of values should be used to estimate parameters, in order to account for uncertainty. Significance testing allows us to make decisions about hypotheses within a particular context.

**Essential Questions** 

How can we make sure that samples are independent?

*Why is it inappropriate to accept a hypothesis as true based on the results of statistical inference testing?* 

## AP Statistics Unit 8 Overview

## Unit Title: Inference for Categorical Data: Chi-Square

## **Unit Summary**

Unit 6 introduced inference for proportions of categorical data. Unit 8 introduces chisquare tests, which can be used when there are two or more categories. Students need to understand how to select from the following tests: the chi-square test for goodness of fit (for a distribution of proportions of one categorical variable in a population), the chi-square test for independence (for associations between categorical variables within a single population), or the chi-square test for homogeneity (for comparing distributions of a categorical variable across populations or treatments). To integrate conceptual understanding, teachers can make connections between frequency tables, conditional probability, and calculating expected counts. The chi-square statistic is introduced to measure the distance between observed and expected counts relative to expected counts.

#### **Approximate Time Needed**

2-3 weeks, Early April

## Unit Foundation

#### **Assessed Competencies**

Knowledge Constructor: Probability and Simulation Knowledge Constructor: Selecting Statistical Methods Critical Thinker: Statistical Argumentation Effective Communicator: Mathematical Communication

#### Assessed Standards

- Determine relative frequencies, proportions, or probabilities using simulation or calculations.
- Describe probability distributions.
- Calculate a test statistic and find a p-value, provided conditions for inference are met.
- Identify null and alternative hypotheses.
- Identify an appropriate inference method for significance tests.
- Interpret statistical calculations and findings to assign meaning or assess a claim.
- Verify that inference procedures apply in a given situation.
- Justify a claim using a decision based on significance tests.

## **Curriculum Framing Questions**

Enduring Understandings

Students will understand that...

Given that variation may be random or not, conclusions are uncertain.

The chi-square distribution may be used to model variation.

Significance testing allows us to make decisions about hypotheses within a particular context.

**Essential Questions** 

How does increasing the degrees of freedom influence the shape of the chi-square distribution? Why is it inappropriate to use statistical inference to justify a claim that there is no association between variables?

## **Data Science & Statistics**